Lecture 6
Chain Reactions & Criticality
First a quick reprise of reaction cross sections
Measuring the diameter of a bowling ball without a ruler

Bale of hay, containing bowling balls of unknown cross-sectional area $\sigma$.

The density of bowling balls is given by $n \text{ [m}^{-3}\text{]}$.

This is the front view, but you really can’t see the bowling balls! They’re hidden inside.
Side view – now let’s blast away with a shotgun
Definition of a cross-section, $\sigma[\text{cm}^2]$

$$N_{\text{scatter}} = N_{\text{in}} \cdot n[\text{cm}^{-3}] \cdot t[\text{cm}] \cdot \sigma[\text{cm}^2]$$

$$\rho_\# \ [\text{cm}^{-2}]$$

The areal density, i.e. the *number* of scattering centers per square centimeter.

The basis for measuring a cross section is that the probability of scattering is the fraction of area occluded by the target nuclei.
Attenuation curves & concept of Mean Free Path (m.f.p.)

\[ N_x = N_0 e^{-\mu x} \]
The Mean Free Path (m.f.p.) is that distance for which the unattenuated (unscattered) beam has been reduced to $1/e = 0.3679$ of its original value.

Examining the expression in the previous viewgraph, the mean-free path, often written $\lambda$, is thus given by $\lambda = 1/\mu$. This begs the question, however, what is $\mu$?

Quite plainly then:

$$\mu \ [\text{cm}^{-1}] = \rho_\# \ [\text{cm}^{-3}] \ \sigma \ [\text{cm}^2]$$

Depends on density & material

Depends on underlying nuclear physics
What are the two isotopes from which one can most readily make either a nuclear reactor or a weapon?

- $^{235}\text{U}$
  - Minority component of natural U (0.7%)
  - Four separation methods: electromagnetic, gaseous diffusion, centrifuge, laser isotope

- $^{239}\text{Pu}$
  - Produced as a by-product in $^{235}\text{U}$ reactors
  - Then can be easily chemically separated
  
  \[
  ^{238}\text{U} + ^{0}\text{n} \rightarrow ^{239}\text{U} \xrightarrow{23.5 \text{ min}} ^{239}\text{Np} \xrightarrow{2.3565 \text{ d}} ^{239}\text{Pu}
  \]
  
Both fission upon capture of slow neutrons (thermal)
Now a review of reactor physics basics
The general concept of a reactor...

A fission event usually produces more than one neutron. Fast neutrons won't efficiently cause fission. The hydrogens in water are nearly equal to neutrons in mass. Collisions with hydrogens are effective in slowing the neutrons.

Water as coolant and moderator flows between fuel rods.
A little better picture
Neutron Multiplication Factor

- $k < 1$: Subcritical
- $k = 1$: Critical
- $k > 1$: Supercritical
- $k \gg 1$: Bomb
The life of each neutron in a reactor is like "Indiana Jones and the Temple of Doom"
4-Factor Formula – fuel of infinite extent

\[ k_\infty = \eta \varepsilon p f \]

\( \eta \) = Number of neutrons emitted per neutron captured in the fuel

\( \varepsilon \) = Fraction of fast neutrons that induce fission

\( p \) = Fraction of neutrons surviving resonance region

\( f \) = Fraction of thermal neutrons captured by fuel
The fission cross sections as they were known in 1942 (from Serber’s “LANL Primer”). Not too bad!

(thermal) log neutron energy in EV.

More modern measurements
Neutron capture in the Resonance Region

$^{56}\text{Fe}(n,\text{tot})$
Infinite in extent
A fission event that produces 4 neutrons.
Finite in extent – Just what you see here
Good bye fast neutron!

Good bye slow neutron!
6-Factor Formula – Fuel of *finite* extent

\[
k = \eta \varepsilon p f L_f L_t
\]

\[L_f = \text{Probability fast neutron does not escape the assembly}\]

\[L_f = \text{Probability slow neutron does not escape the assembly}\]
So total MASS is important
Ahah!
Not only is **MASS** important,
But **DENSITY** is important
Same total mass, but now extended in both directions
So what’s important now?

MASS,
DENSITY,
SHAPE

But hold on, one more coming …
Neutron Reflector (sometimes dual purpose as ‘tamper’)
At long last:

MASS,
DENSITY,
SHAPE,
NEUTRON REFLECTION
Fission cross section

Experimental results from fission studies