Lecture 2

Atoms, Nuclei & the Curve of Binding energy

Toolkit – Lecture #1

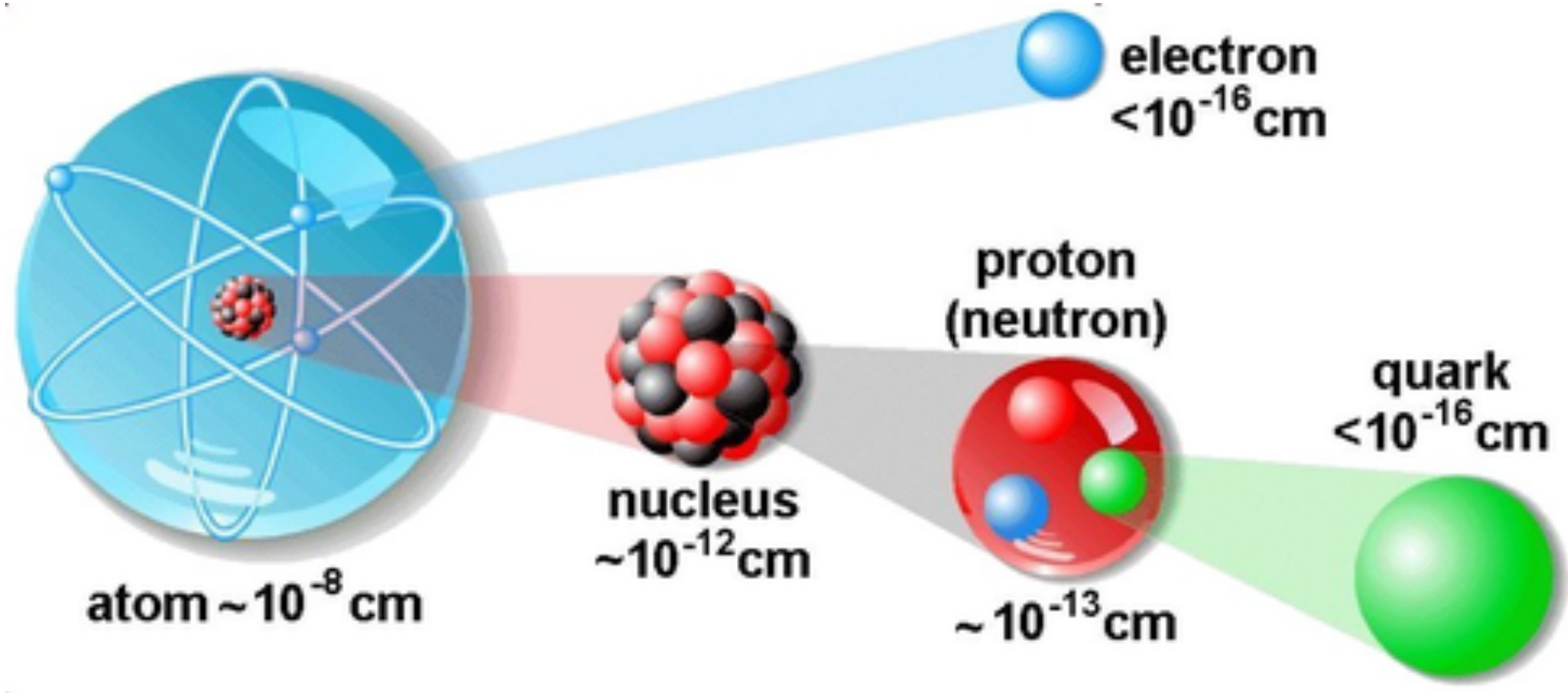
- 1 kilotonne = 4.184×10^{12} J (or 1 gm TNT = 1 kilocalorie, kcal)
- 1 eV = 1.602×10^{-19} J (or 1 J = 6.24×10^{18} eV)
- 1 e = 1.602×10^{-19} Coulomb (or 1 Coulomb = 6.24×10^{18} e)
- $e^2 = 1.44$ MeV $\cdot$ fm (1 fm = 1 fermi = 10^{-15} meter)
- $\hbar c = 197.3$ MeV $\cdot$ fm ($\hbar = h / 2\pi$)
- 1 mole = 6.02×10^{23} (Avogadro's number)
- $c = 3 \times 10^8$ m/sec (Speed of light in vacuum)

Experiments on a pond at Clapham

"I fetched out a cruet of oil and dropped a little of it on the water. I saw it spread itself with surprising swiftness upon the surface… Though not more than a teaspoonful, produced an instant calm over a space several yards square which spread amazingly and extended itself gradually till it reached the lee side, making all that quarter of the pond, perhaps half an acre, as smooth as a looking glass."

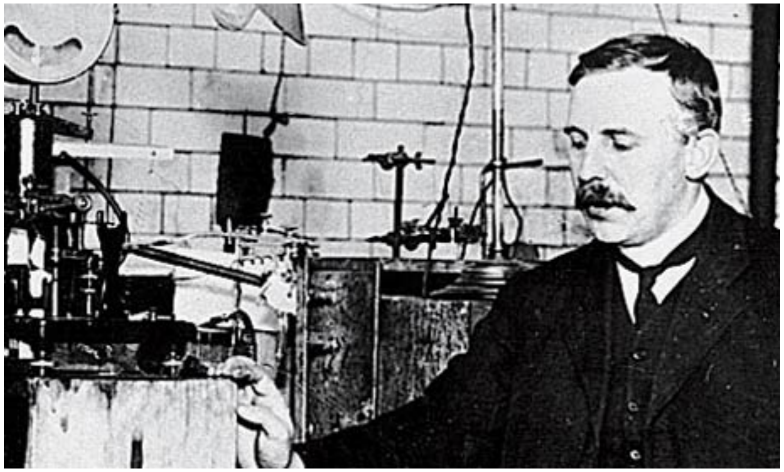
"After this I contrived to take with me, whenever I went into the country, a little oil in the upper hollow joint of my bamboo cane, with which I might repeat the experiment and I found it constantly to succeed."

Size of the nucleus & the entire atom

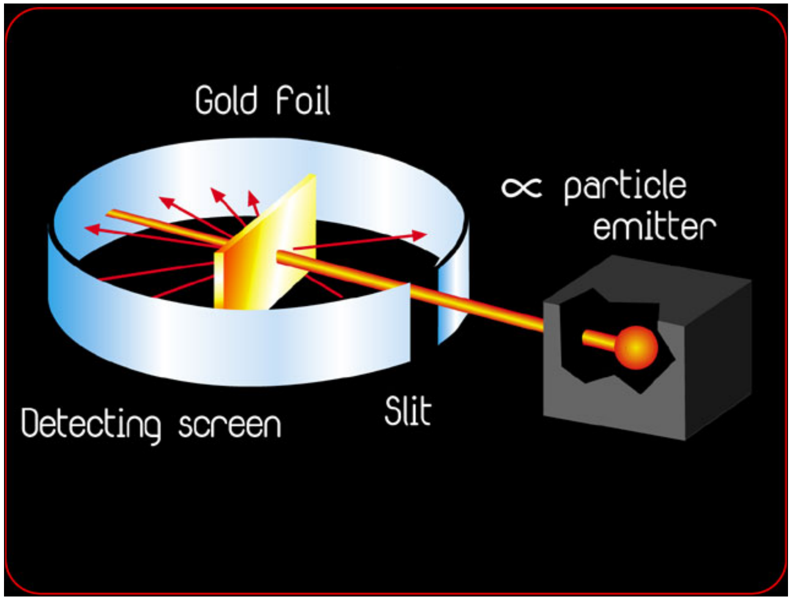


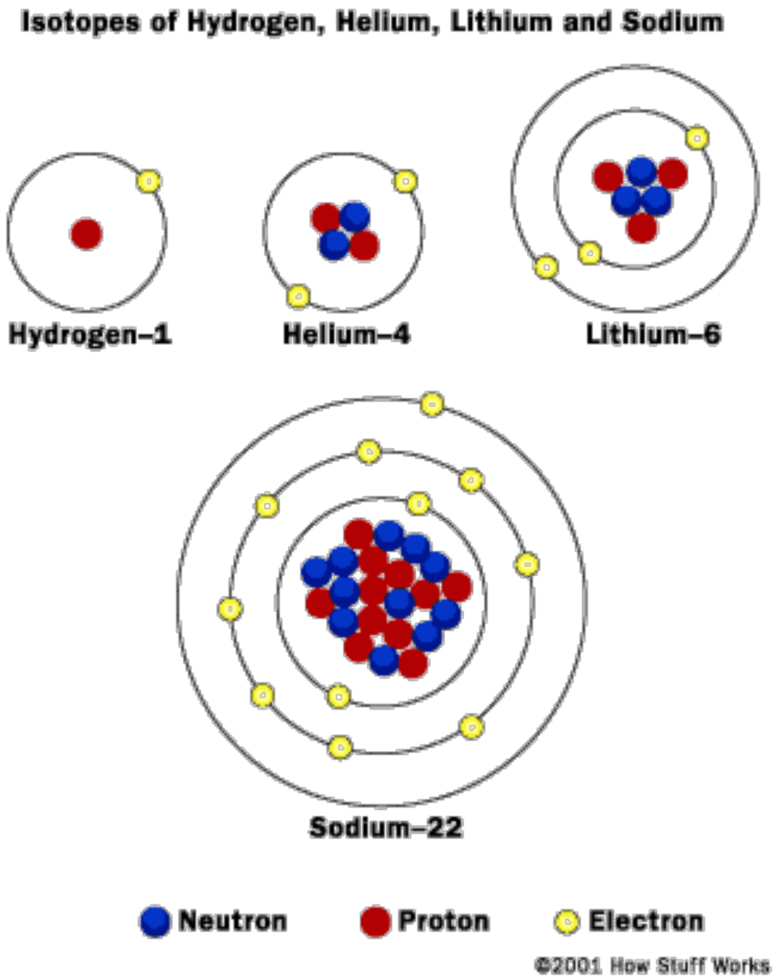
If the nucleus were scaled up to be the size of a marble (~ 1 cm), then the cloud of atomic electrons would extend over to McLaughlin Hall (about 100 m away) !

Ernest Rutherford's scattering experiment (>100 years ago)



"It's as if you fired a 15-inch Naval shell at a piece of tissue paper, and it came back!"
Ernest Rutherford





Notation

A = Mass Number

= Number of protons + neutrons

= Z + N

A

$$^{27}_{13}\mathrm{Al}$$

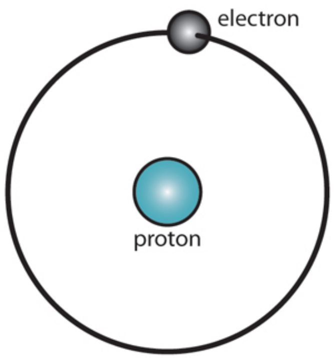
Z

(but usually not written)

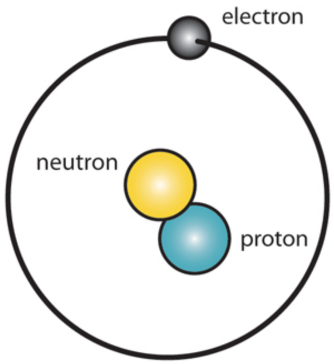
Z = Atomic Number

= Number of protons

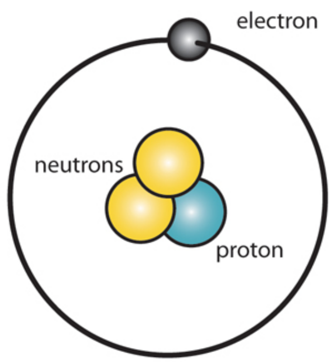
Examples (apologies for the cheesy graphics)



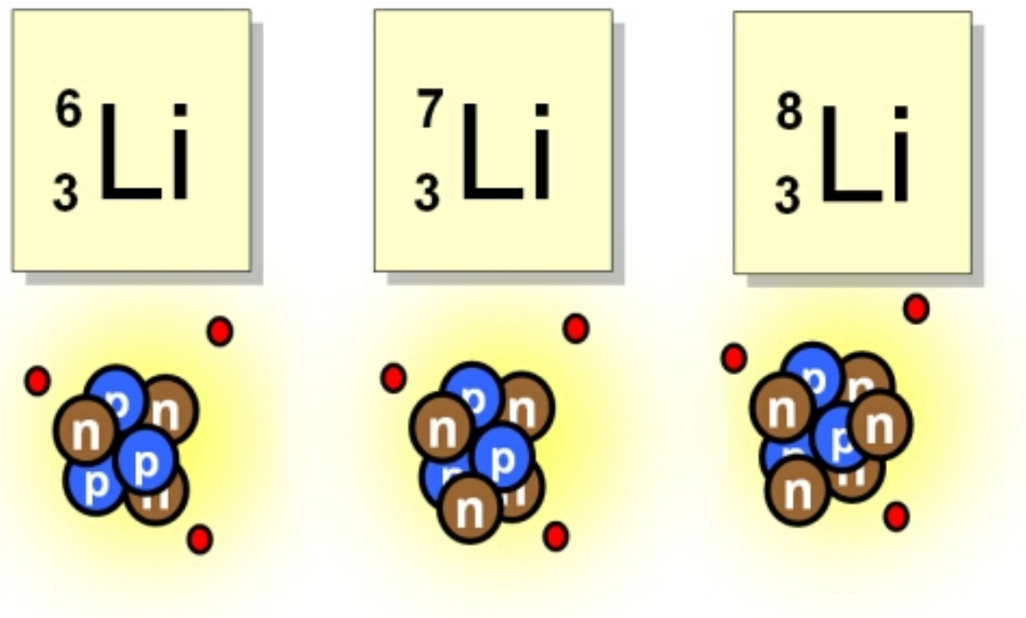
H hydrogen



^{2}H or D deuterium

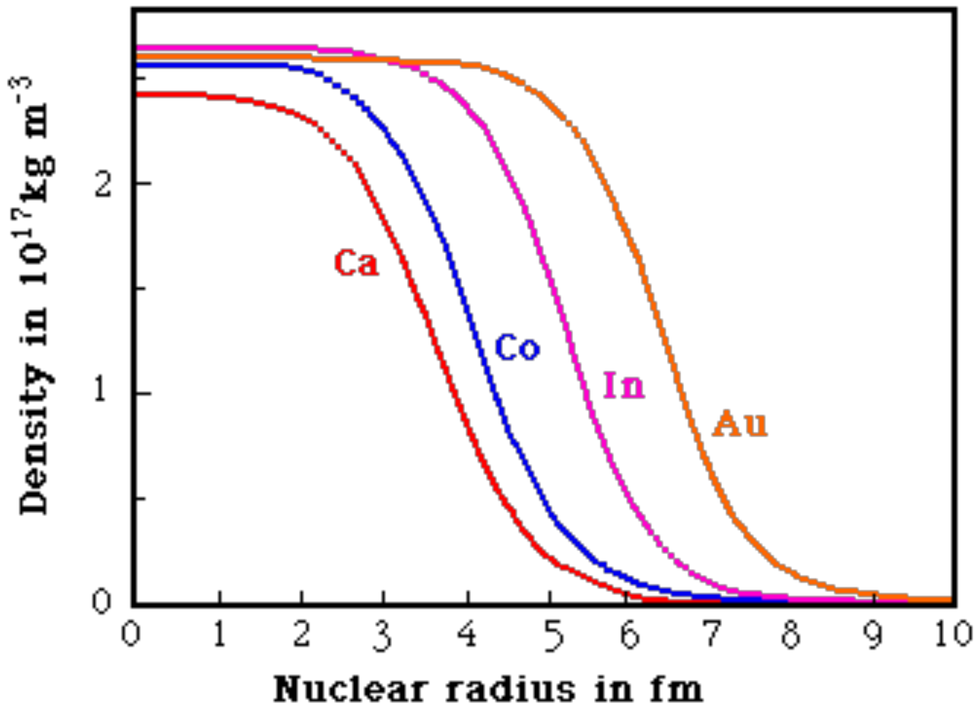


^{3} H or T tritium



The nucleus…

- Is very small: $R \approx r_0 \cdot A^{1/3}$
  - $r_0 \approx$ (1.2-1.4) fm
- Reasonably sharp edge
- Nuclear force is strong & short ranged
  - Range of force ~ 1 fm
- $Z \approx N$, for light nuclei
  - But heavy are neutron-rich
- Can have many isotopes for each element



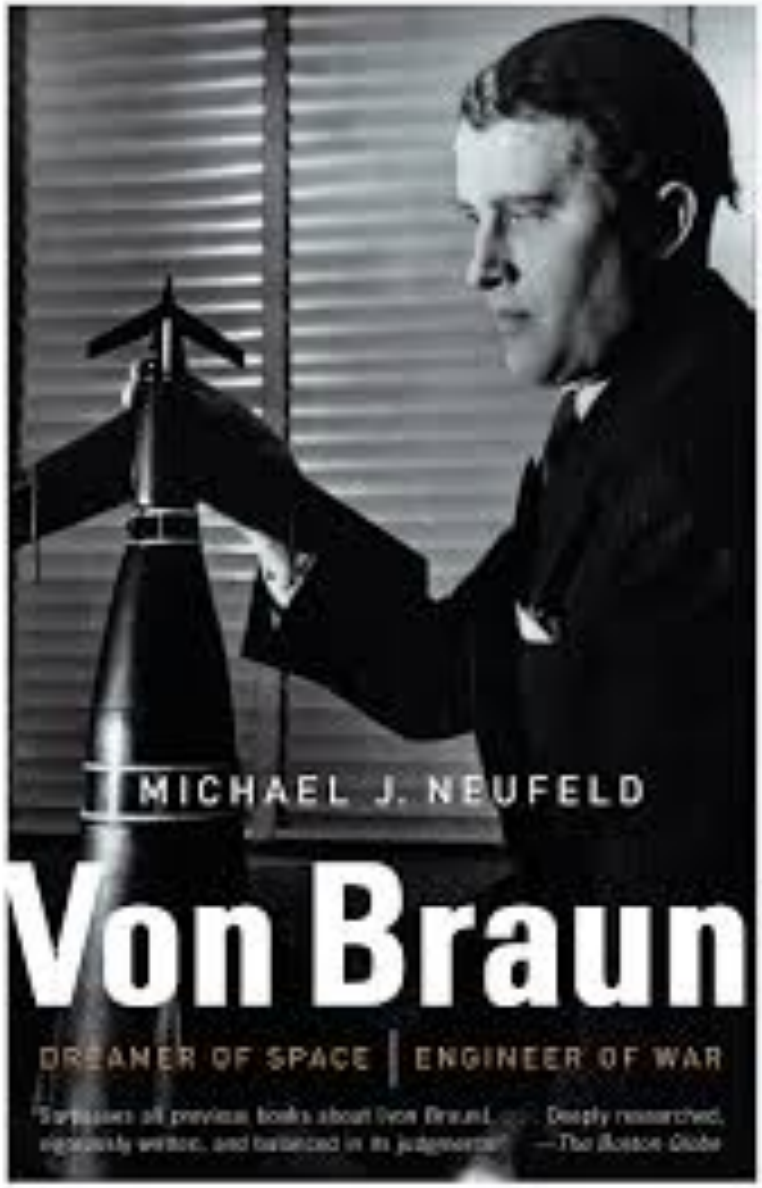
Masses: e, p, n, ^{1}H, u … first in MKS

- $m_e = 9.109 \times 10^{-31}$ kg
- $m_p = 1.673 \times 10^{-27}$ kg (bare proton, *not* a hydrogen atom)
- $m_n = 1.675 \times 10^{-27}$ kg
- m_u = (1/12) x (Mass of ^{12}C *atom*)

  $= 1.661 \times 10^{-27}$ kg (a.k.a. "a.m.u.", or "u")

Note the proton and neutron are almost the same mass.
Note proton is roughly 2000 times heavier than an electron.

On the missile delivery side of the equation …



Atomic masses

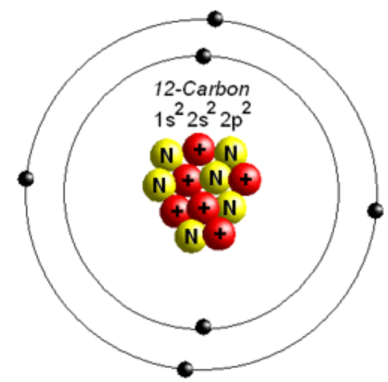
$m_pc^2 = 938.272$ MeV

$m_nc^2 = 939.565$ MeV

$m_ec^2 = 0.511$ MeV

1 Atomic Mass Unit (amu) is defined to be 1/12 of the mass of a ^{12}C atom :
note it includes 6 electrons!

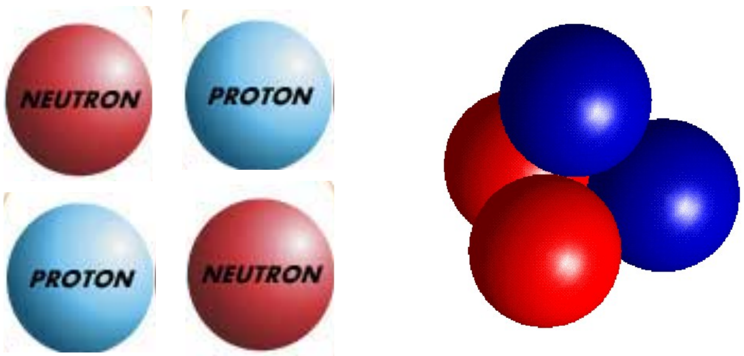
Mass 1 a.m.u. $= m_uc^2 = 931.494$ MeV $= 1/12 \times$

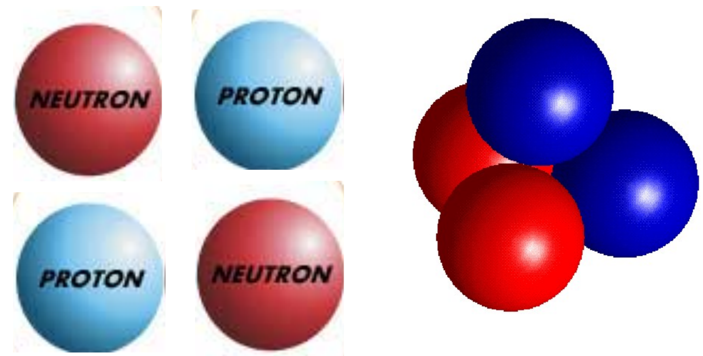


What you often see in the Nuclear Data Tables & Charts is the *Mass Excess:*

$$\Delta_{(Z,N)} = \left(M_{(Z,N)}\ [\text{amu}] - A \right) \times m_uc^2 \quad [\text{MeV}]$$

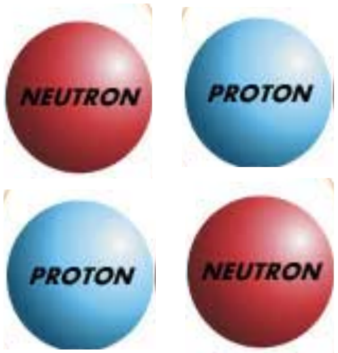
Question - which weighs more?

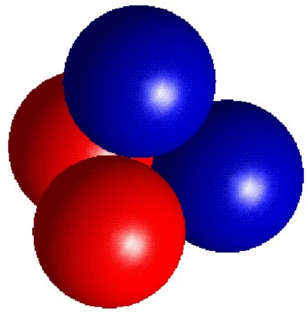




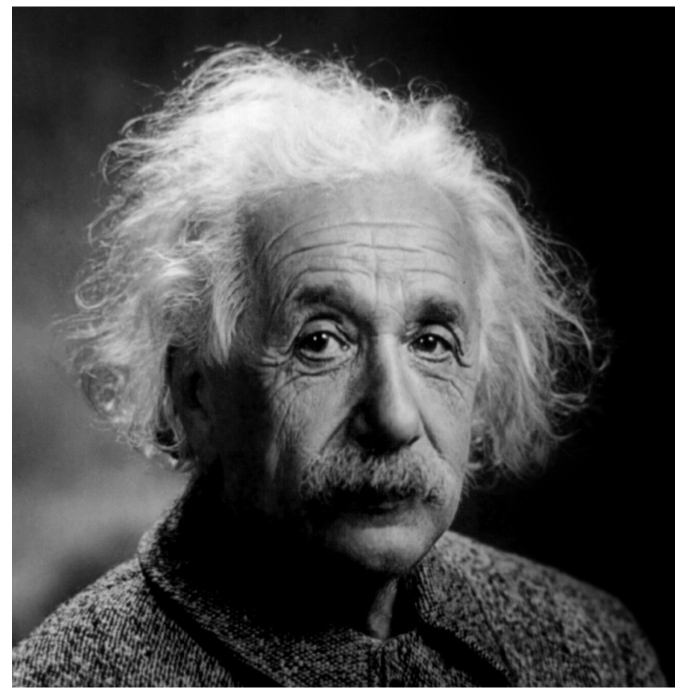
Is it this? or Is it this?

It's this case!





$$E = mc^2$$



$$Q = \left(\sum_{\text{initial}} m_i\right)c^2 - \left(\sum_{\text{final}} m_i\right)c^2$$

Other conventions for mass

- Mass in a.m.u. (a pure number), i.e.:
  - Mass ^{1}H = 1.007825
  - Mass ^{4}He = 4.00260
  - Mass ^{12}C = 12.000000
  - Mass ^{16}O = 15.99491
- Binding Energy (B.E.) = $Z \cdot M_H + N \cdot M_N - M(Z,N)$
  - Units usually in [MeV]
  - You're already thinking in nuclear units; c^2 is implied!

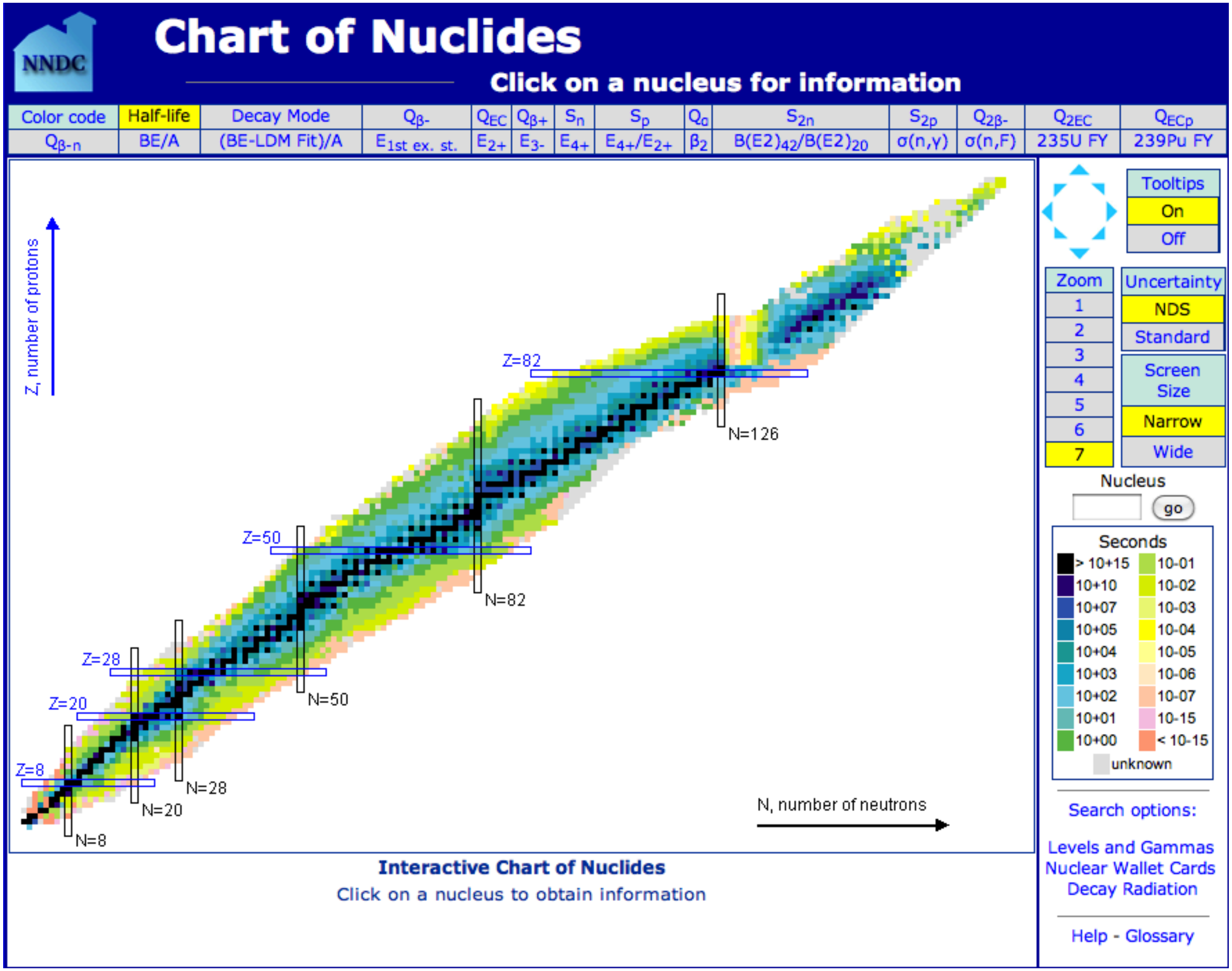
Table 23 MASSES OF SOME NUCLIDES

Element	Symbol	Atomic mass* (u)**	Element	Symbol	Atomic mass* (u)**
hydrogen	$^{1}_{1}H$	1.007825	sodium	$^{23}_{11}Na$	22.9898
deuterium	$^{2}_{1}H$	2.0140	magnesium	$^{24}_{12}Mg$	23.98504
helium	$^{3}_{2}He$	3.01603		$^{25}_{12}Mg$	24.98584
	$^{4}_{2}He$	4.00260		$^{26}_{12}Mg$	25.98259
lithium	$^{6}_{3}Li$	6.01512	sulfur	$^{32}_{16}S$	31.97207
	$^{7}_{3}Li$	7.01600	chlorine	$^{35}_{17}Cl$	34.96885
beryllium	$^{6}_{4}Be$	6.0197		$^{37}_{17}Cl$	36.96590
	$^{8}_{4}Be$	8.0053	potassium	$^{39}_{19}K$	38.96371
	$^{9}_{4}Be$	9.01218		$^{41}_{19}K$	40.96184
boron	$^{10}_{5}B$	10.0129	krypton	$^{95}_{36}Kr$	94.9
	$^{11}_{5}B$	11.00931	molybdenum	$^{100}_{42}Mo$	99.9076
carbon	$^{12}_{6}C$	12.00000	silver	$^{107}_{47}Ag$	106.90509
	$^{13}_{6}C$	13.00335		$^{109}_{47}Ag$	108.9047
nitrogen	$^{12}_{7}N$	12.0188	tellurium	$^{137}_{52}Te$	136.91
	$^{14}_{7}N$	14.00307	barium	$^{138}_{56}Ba$	137.9050
	$^{15}_{7}N$	15.00011	lead	$^{214}_{82}Pb$	213.9982
oxygen	$^{16}_{8}O$	15.99491	bismuth	$^{214}_{83}Bi$	213.9972
	$^{17}_{8}O$	16.99914	polonium	$^{218}_{84}Po$	218.0089
	$^{18}_{8}O$	17.99916	radon	$^{222}_{86}Rn$	222.0175
fluorine	$^{19}_{9}F$	18.99840	radium	$^{228}_{88}Ra$	228.0303
neon	$^{20}_{10}Ne$	19.99244	thorium	$^{232}_{90}Th$	232.0382
	$^{22}_{10}Ne$	21.99138	uranium	$^{235}_{92}U$	235.0439
				$^{238}_{92}U$	238.0508
			plutonium	$^{239}_{94}Pu$	239.0522

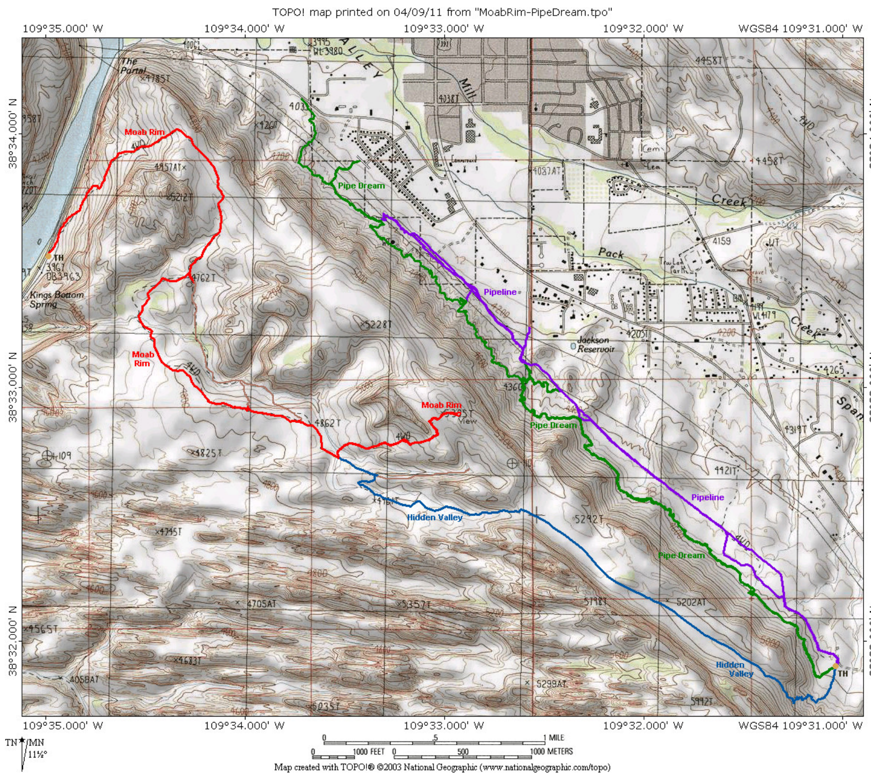
*Atomic mass of neutral atom is given.

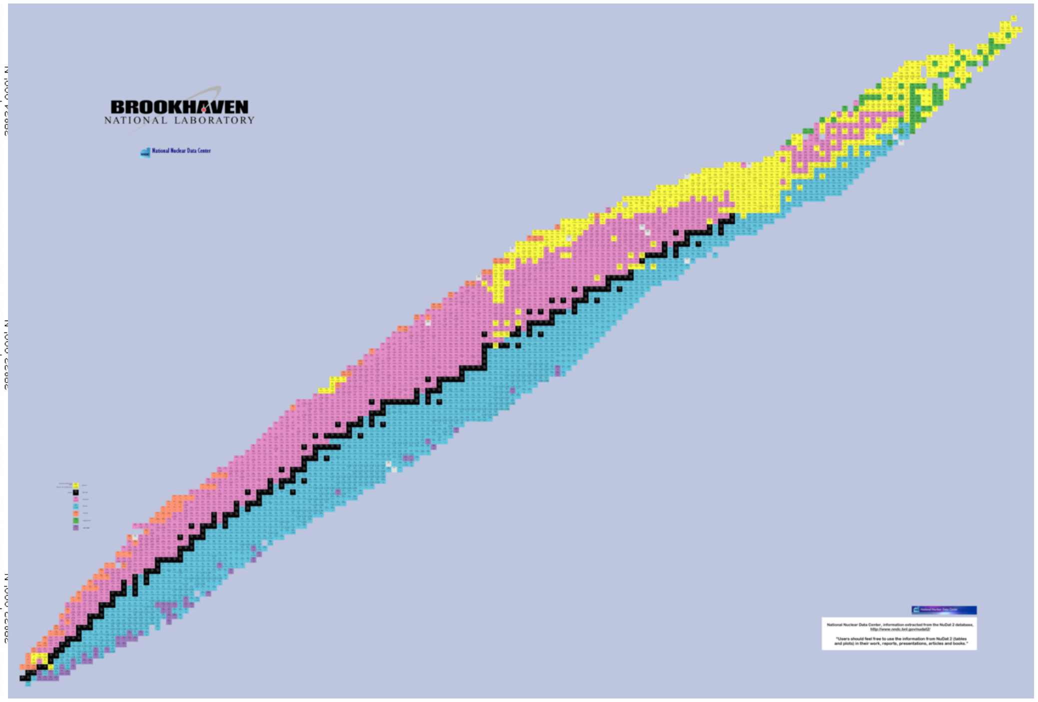
**1 atomic mass unit $(u) = 1.6605655 \times 10^{-27}$ kg.

http://www.nndc.bnl.gov/chart/ - very useful!

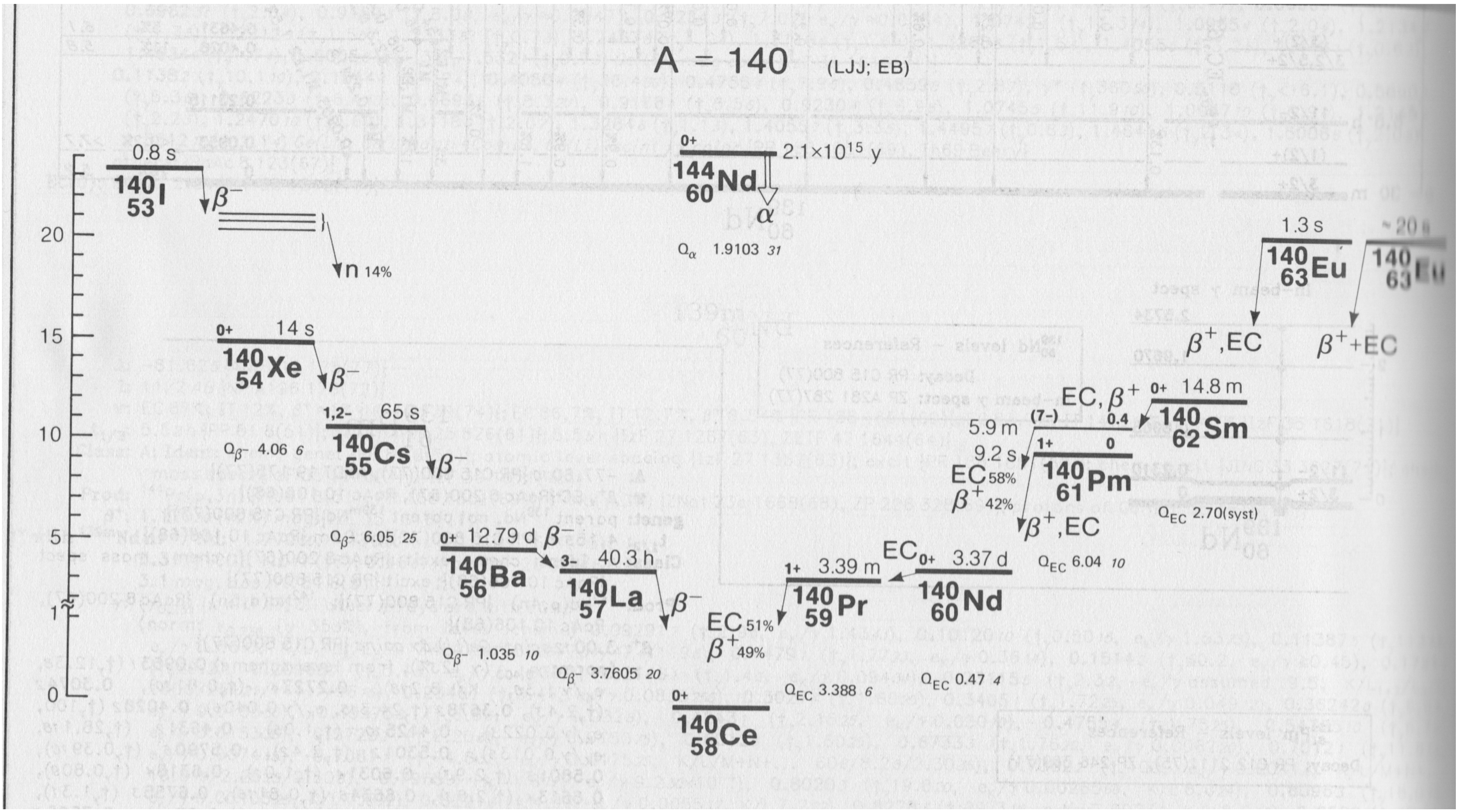


Analogy: Think of the Chart of the Nuclides as a topo map of a valley (below Moab Rim, Utah)



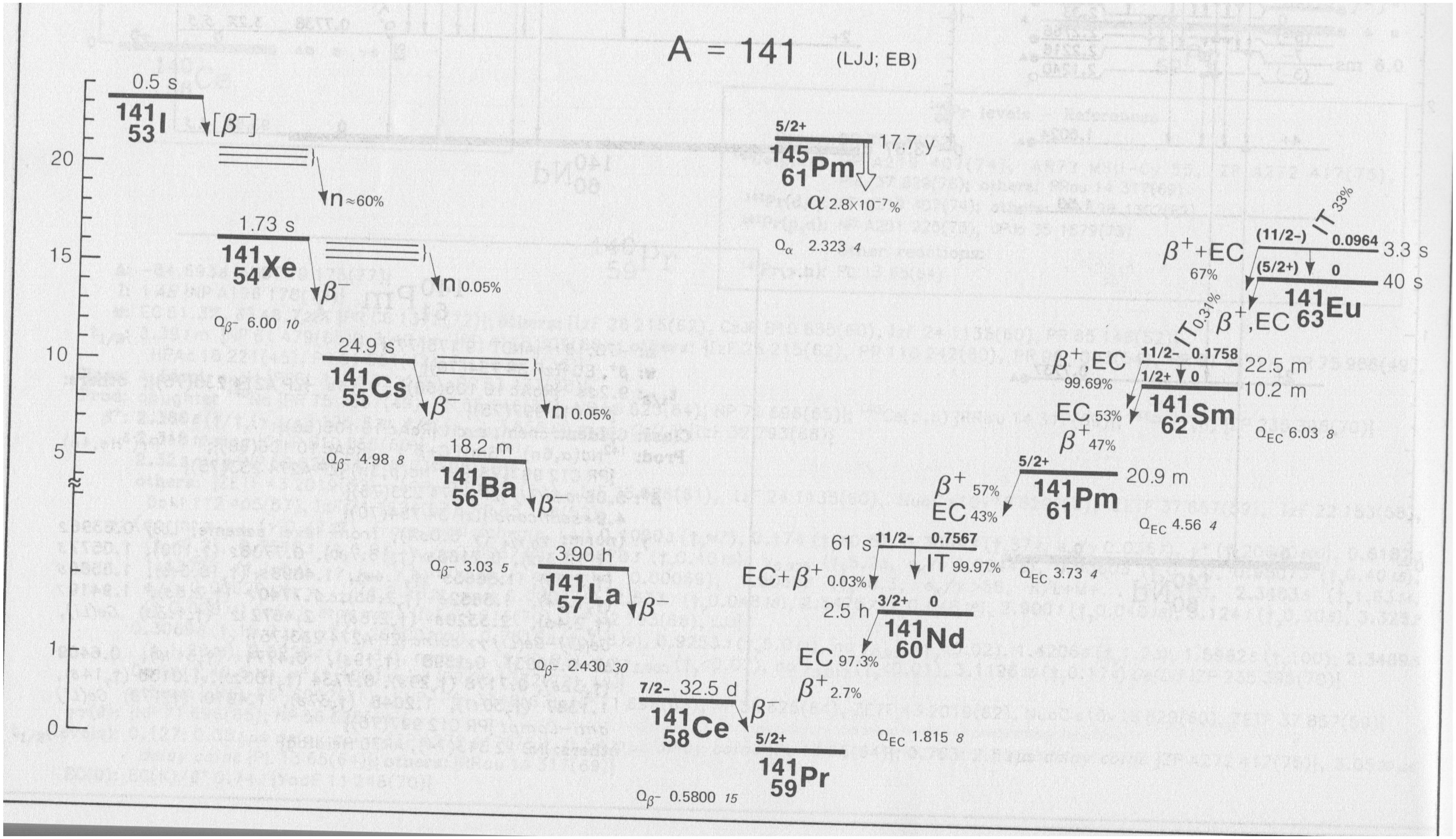


A = 140



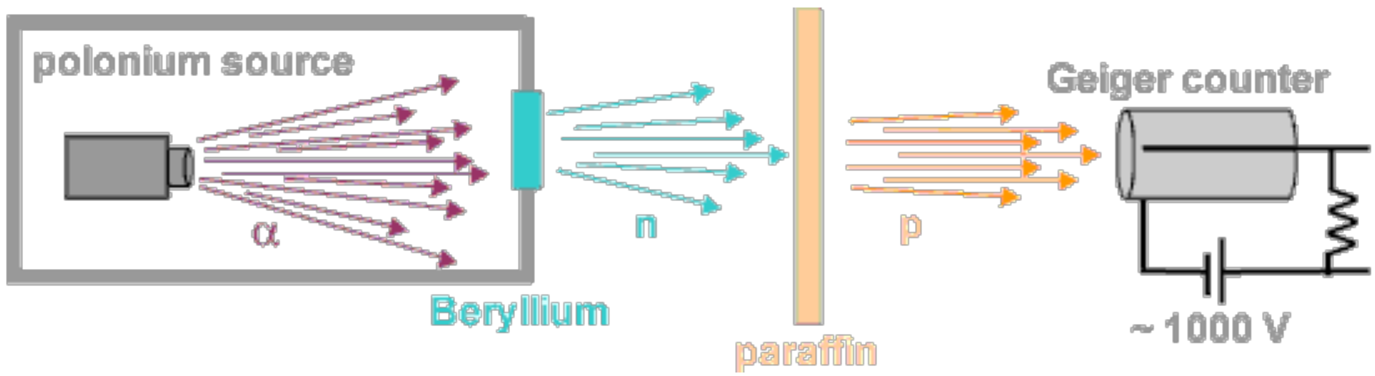
A = 140 (LJJ; EB)
0.8 s
140 53 I
β⁻
n 14%
0+ 14 s
140 54 Xe
β⁻
1,2- 65 s
140 55 Cs
β⁻
Qβ- 4.06 6
Qβ- 6.05 25
0+ 12.79 d β⁻
140 56 Ba
Qβ- 1.035 10
3- 40.3 h
140 57 La
β⁻
Qβ- 3.7605 20
0+
140 58 Ce
1+ 3.39 m
140 59 Pr
EC 51%
β+ 49%
QEC 3.388 6
EC 0+ 3.37 d
140 60 Nd
QEC 0.47 4
0+
144 60 Nd
2.1×10^15 y
α
Qα 1.9103 31
5.9 m (7-) 0.4
9.2 s 1+ 0
140 61 Pm
EC 58%
β+ 42%
β+,EC
QEC 6.04 10
EC, β+ 0+ 14.8 m
140 62 Sm
QEC 2.70(syst)
1.3 s
140 63 Eu
β+,EC
~20 s
140 63 Eu
β+ +EC
20
15
10
5
1
0

A = 141


A = 141 (LJJ; EB)
0.5 s
141 53 I
[β⁻]
n ≈60%
1.73 s
141 54 Xe
β⁻
n 0.05%
Qβ- 6.00 10
24.9 s
141 55 Cs
β⁻
n 0.05%
Qβ- 4.98 8
18.2 m
141 56 Ba
β⁻
Qβ- 3.03 5
3.90 h
141 57 La
β⁻
Qβ- 2.430 30
7/2- 32.5 d
141 58 Ce
β⁻
5/2+
141 59 Pr
Qβ- 0.5800 15
5/2+ 17.7 y
145 61 Pm
α 2.8×10⁻⁷%
Qα 2.323 4
61 s 11/2- 0.7567
IT 99.97%
EC+β⁺ 0.03%
2.5 h 3/2+ 0
141 60 Nd
EC 97.3%
β⁺ 2.7%
QEC 1.815 8
QEC 3.73 4
5/2+ 20.9 m
141 61 Pm
β⁺ 57%
EC 43%
QEC 4.56 4
β⁺,EC 99.69%
11/2- 0.1758 22.5 m
1/2+ 0 10.2 m
IT 0.31%
141 62 Sm
EC 53%
β⁺ 47%
QEC 6.03 8
β⁺+EC 67%
(11/2-) 0.0964 3.3 s
IT 33%
(5/2+) 0 40 s
141 63 Eu
β⁺,EC

Let's calculate a Q-value of a reaction!

Let's do the reaction by which Chadwick discovered the neutron



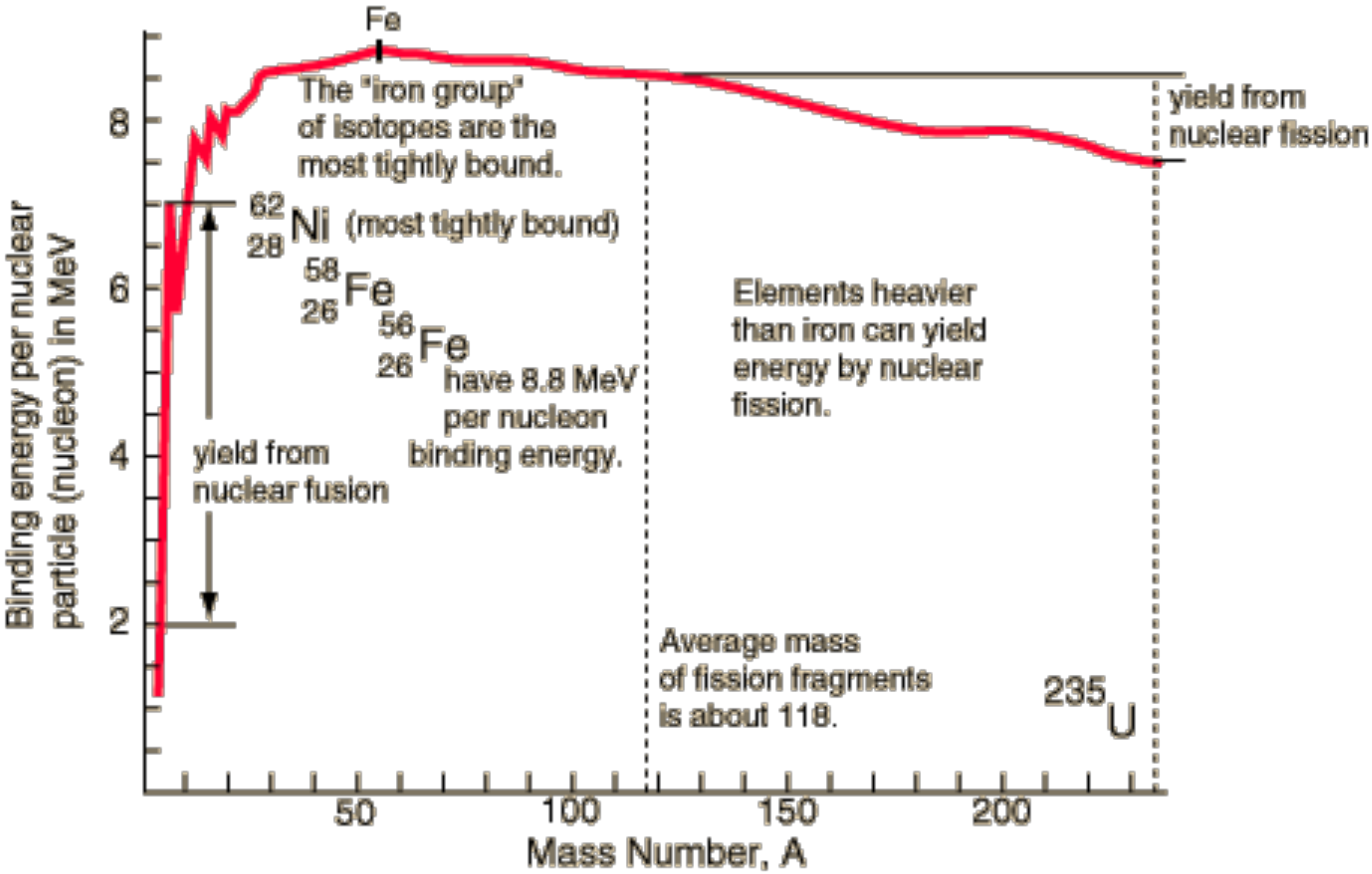
$$^{4}\mathrm{He} + {}^{9}\mathrm{Be} \rightarrow {}^{12}\mathrm{C} + n$$

$$Q = M(^{4}\mathrm{He}) + M(^{9}\mathrm{Be}) - M(^{12}\mathrm{C}) - M(n)$$

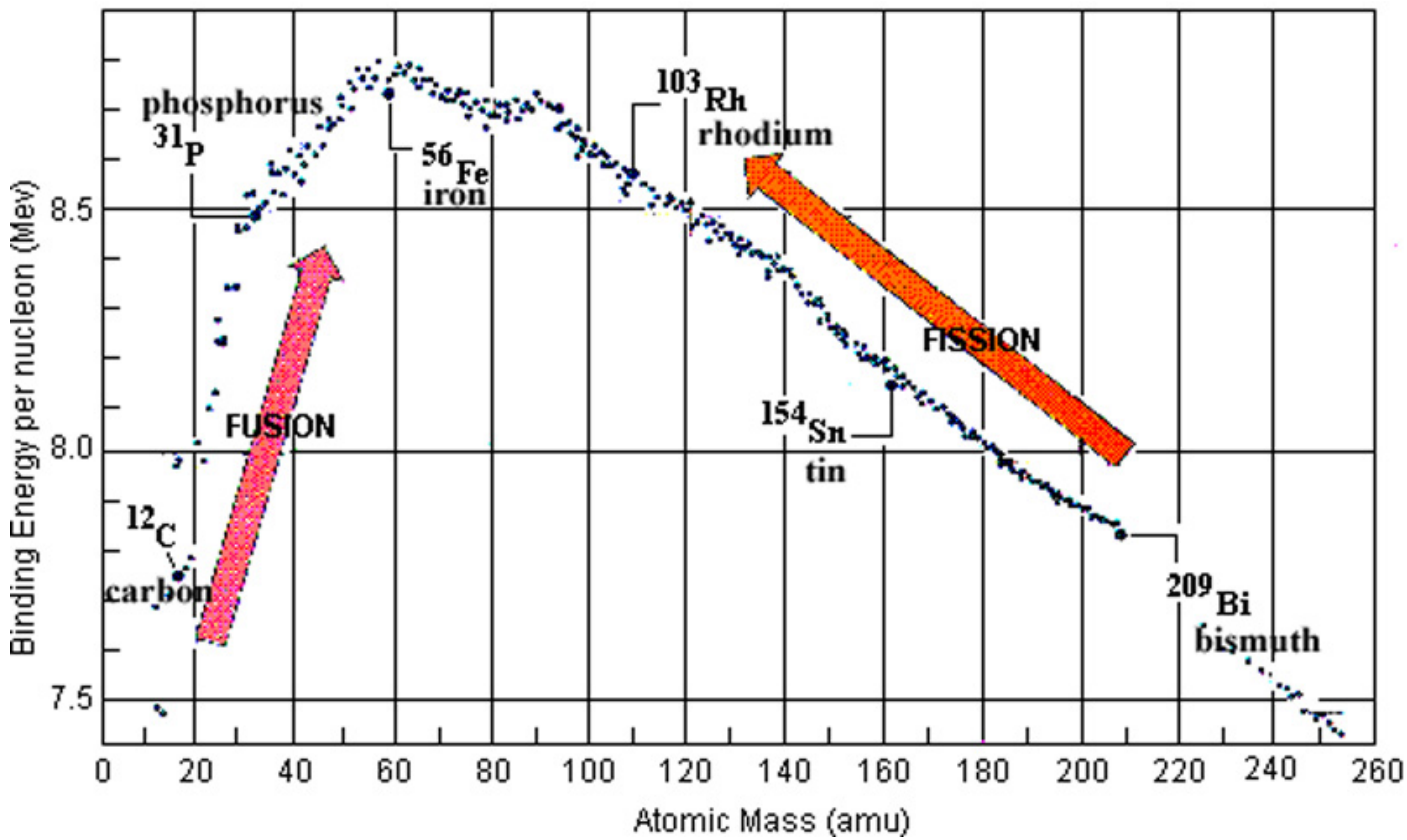
$$= (4.00260 + 9.01218 - 12.00000 - 1.008665) \times 931.494\ \mathrm{MeV}$$

$$= 0.006115 \times 931.494\ \mathrm{MeV} = +5.7\ \mathrm{MeV}\ !$$

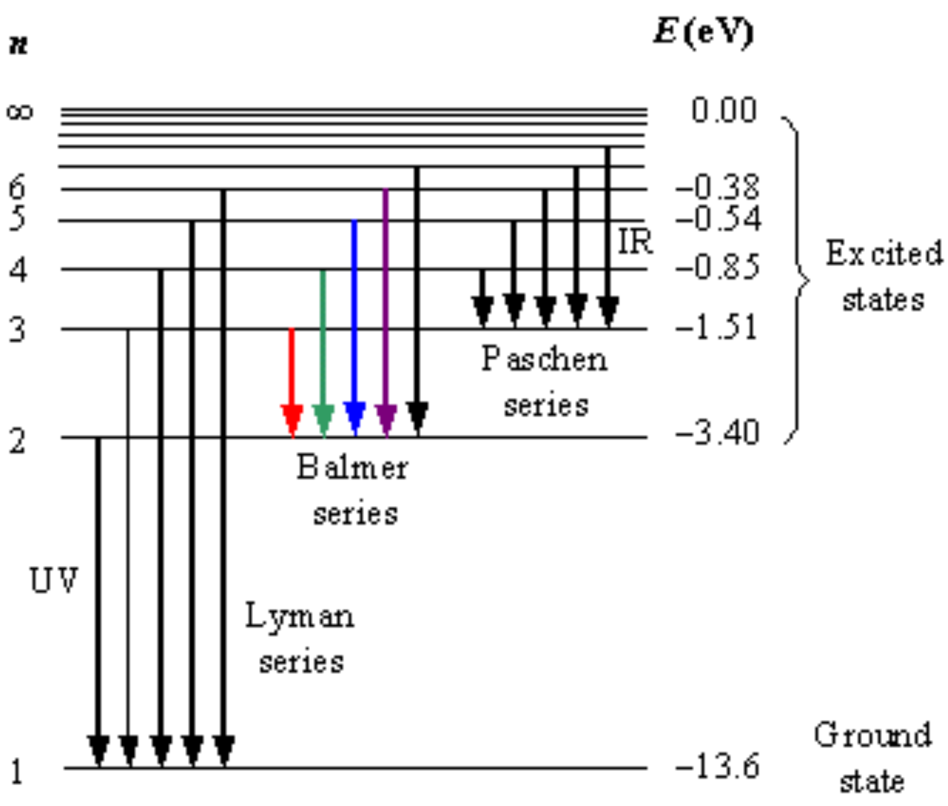
The curve of binding energy



This is the "money slide" !
The whole nuclear power game is on this plot !

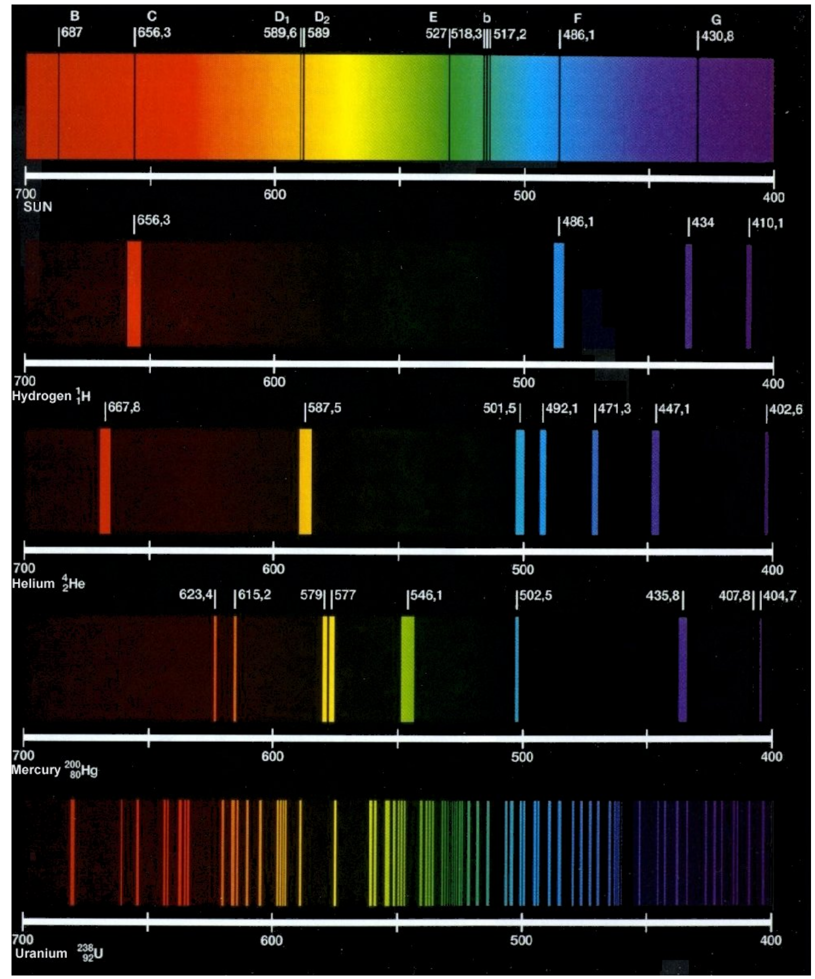


Atoms (their electronic structure) can only exist in discrete energy states



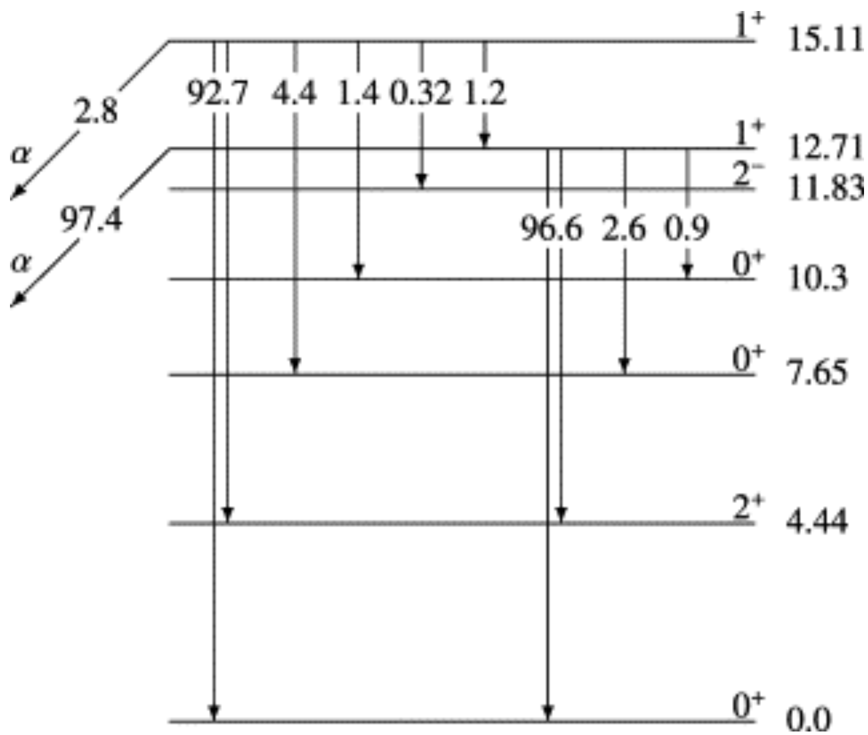
Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

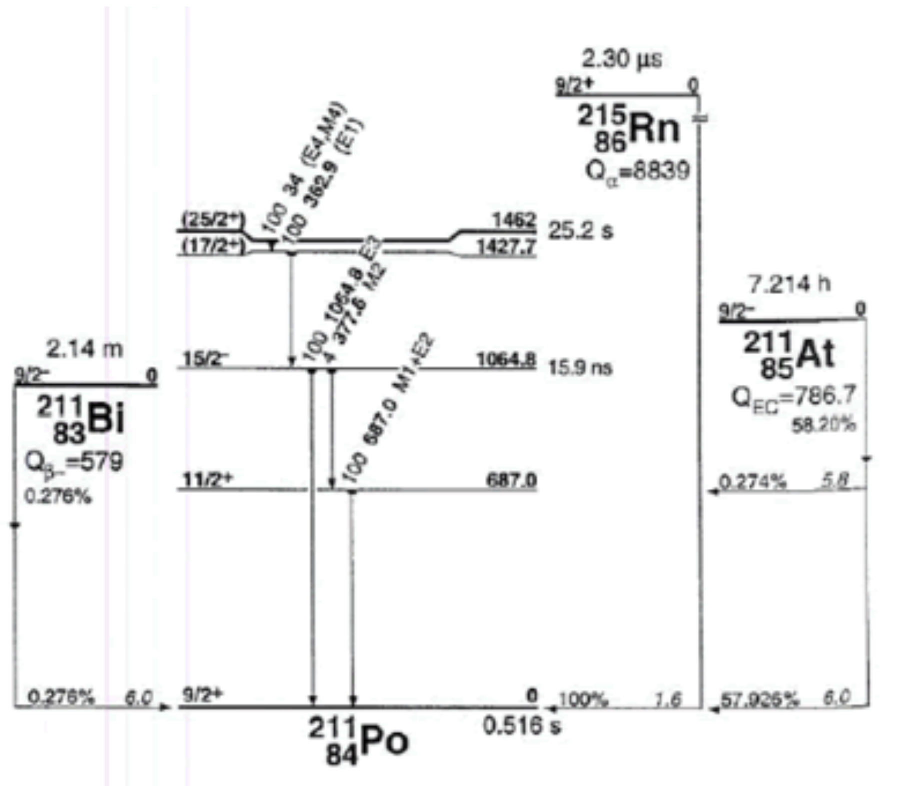
Hydrogen level diagram



Spectral lines, i.e. transitions between levels

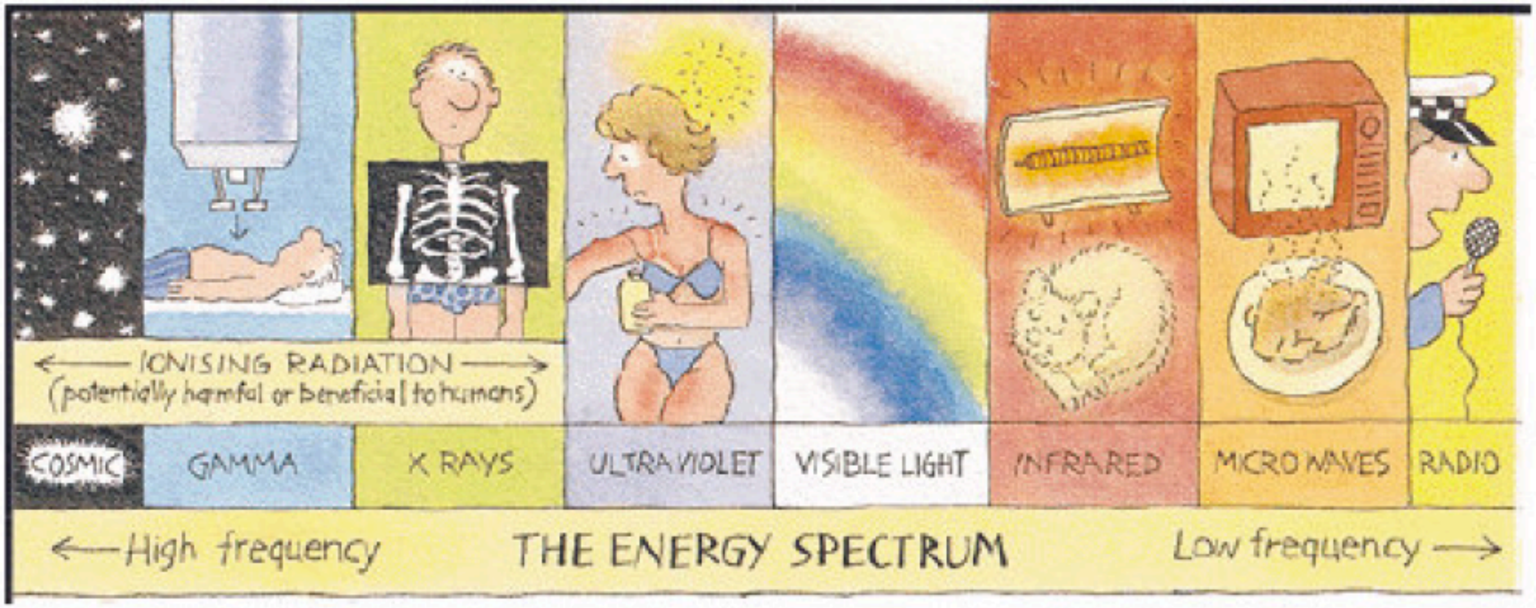
Nuclei have their own structure, and can only exist in discrete energy states, too



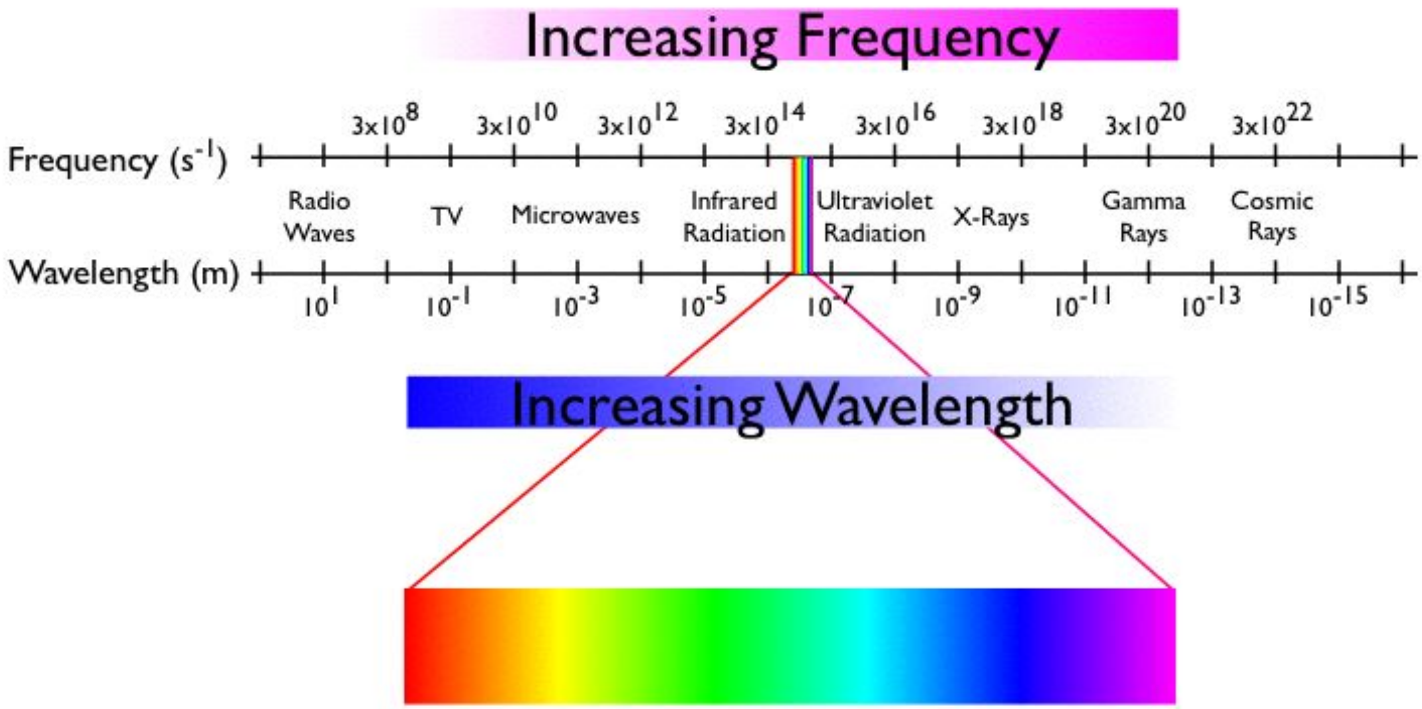


Carbon-12 (energy levels in MeV)

Polonium-211 (energy levels in keV)



IONISING RADIATION
(potentially harmful or beneficial to humans)
COSMIC
GAMMA
X RAYS
ULTRA VIOLET
VISIBLE LIGHT
INFRARED
MICRO WAVES
RADIO
High frequency
THE ENERGY SPECTRUM
Low frequency



Increasing Frequency
Frequency (s^{-1})
$3x10^8$ $3x10^{10}$ $3x10^{12}$ $3x10^{14}$ $3x10^{16}$ $3x10^{18}$ $3x10^{20}$ $3x10^{22}$
Radio Waves
TV
Microwaves
Infrared Radiation
Ultraviolet Radiation
X-Rays
Gamma Rays
Cosmic Rays
Wavelength (m)
10^1 10^{-1} 10^{-3} 10^{-5} 10^{-7} 10^{-9} 10^{-11} 10^{-13} 10^{-15}
Increasing Wavelength

Tool-kit – Lecture #2

$m_e c^2 = 0.511$ MeV
($m_e = 9.109 \times 10^{-31}$ kg)

$m_p c^2 = 938.272$ MeV
($m_p = 1.673 \times 10^{-27}$ kg)

$m_n c^2 = 939.565$ MeV
($m_n = 1.675 \times 10^{-27}$ kg)

$m_u c^2 = 931.494$ MeV
(= 1/12 mass of Carbon-12 atom)

Atoms

Sizes ~ 1 Å (Å = 10^{-10} m)
Energies ~ 1 eV – 10 keV

Nuclei

Sizes ~ 1-10 fm (fm =10^{-15}m)
Energies ~ 100 keV – 10 MeV

Radius of nuclei

$R \approx 1.4\, A^{1/3}$ [fm]

What's the value proposition with Nuclear Energy?

Let's compare the 3 "F"s – Fossil, Fission, Fusion

Annual Fuel Requirements for a 1000 MW Power Plant

Coal	2,500,000 ton	250 trains (100 cars each) (160 lbs/sec)	
Oil	11,000,000 barrels	11 super tankers (15 gallons/sec)	
Fission	28 tons UO_2	1.5 rail car load (150 lbs/day)	
Fusion	100 lb. D 150 lb. T (from 340 lb. of lithium-6)	1/2 ton pickup truck	

***Lithium in one laptop battery + 40 litres of water**

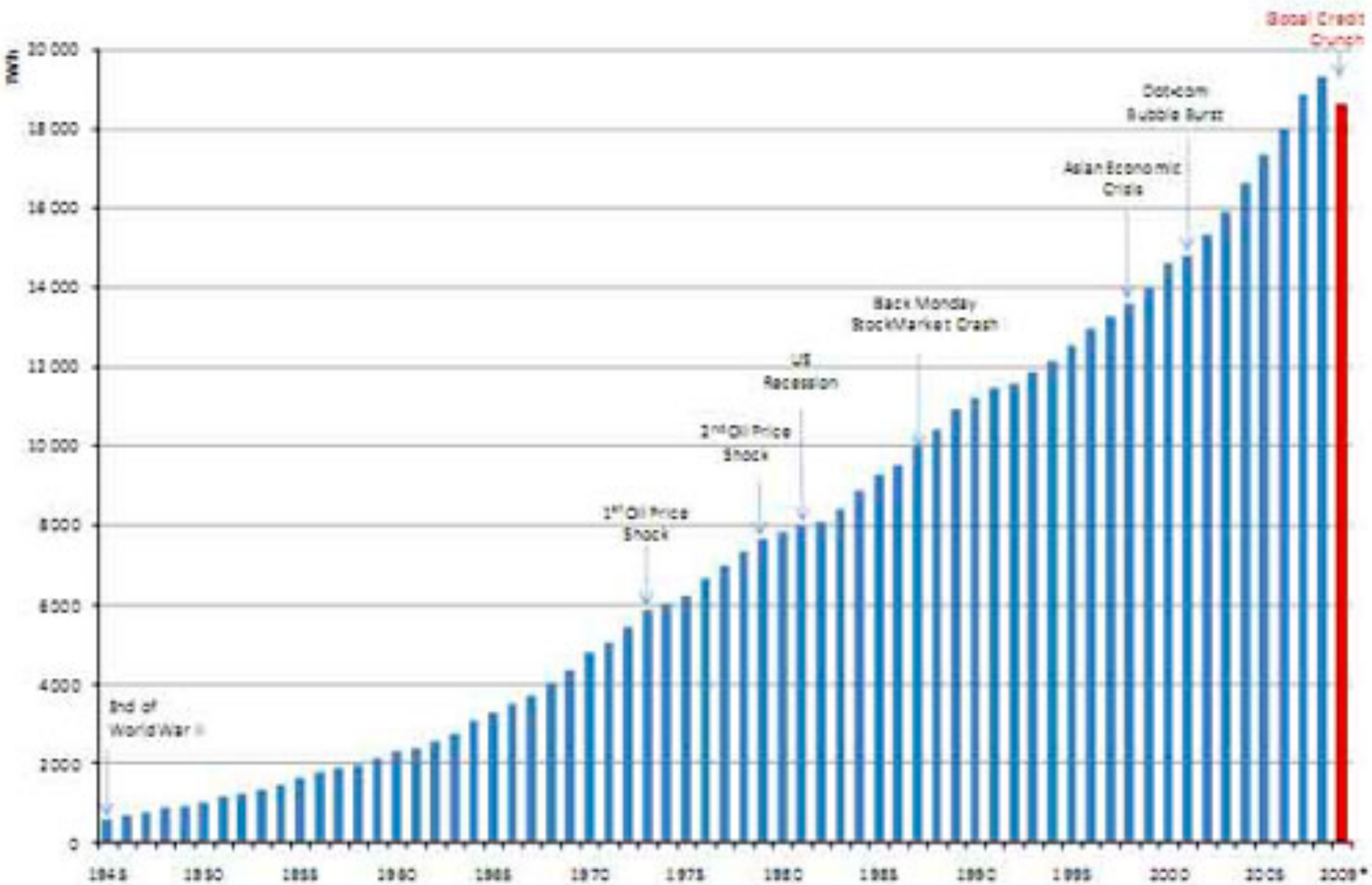
= 200,000 kW-hours in fusion power

= 15 years of per capita US electricity production *intrinsically safe with no CO_2*

*after Sir Christopher Llewelyn-Smith

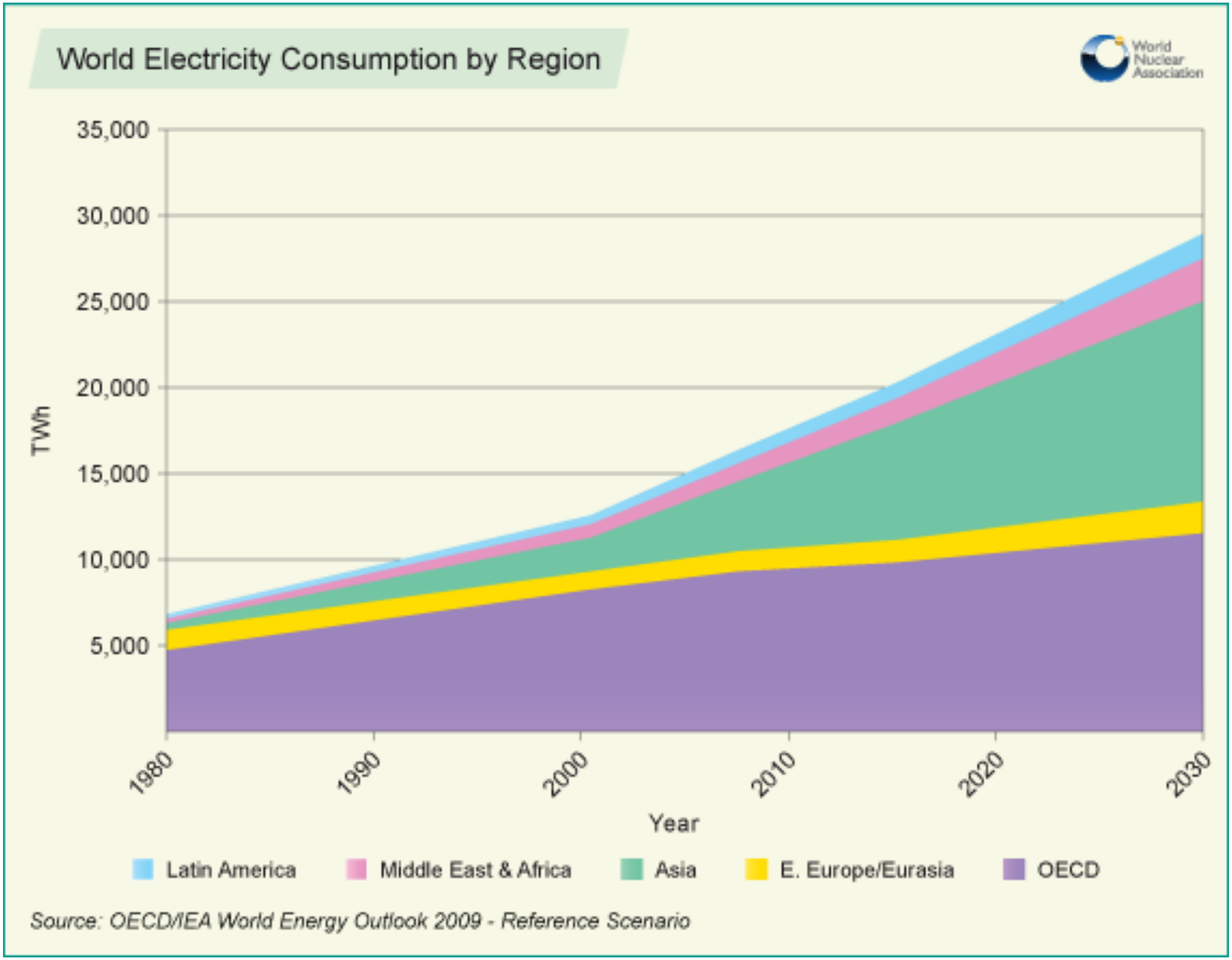
World electricity usage - total

Figure 10: **Historical world electricity consumption**

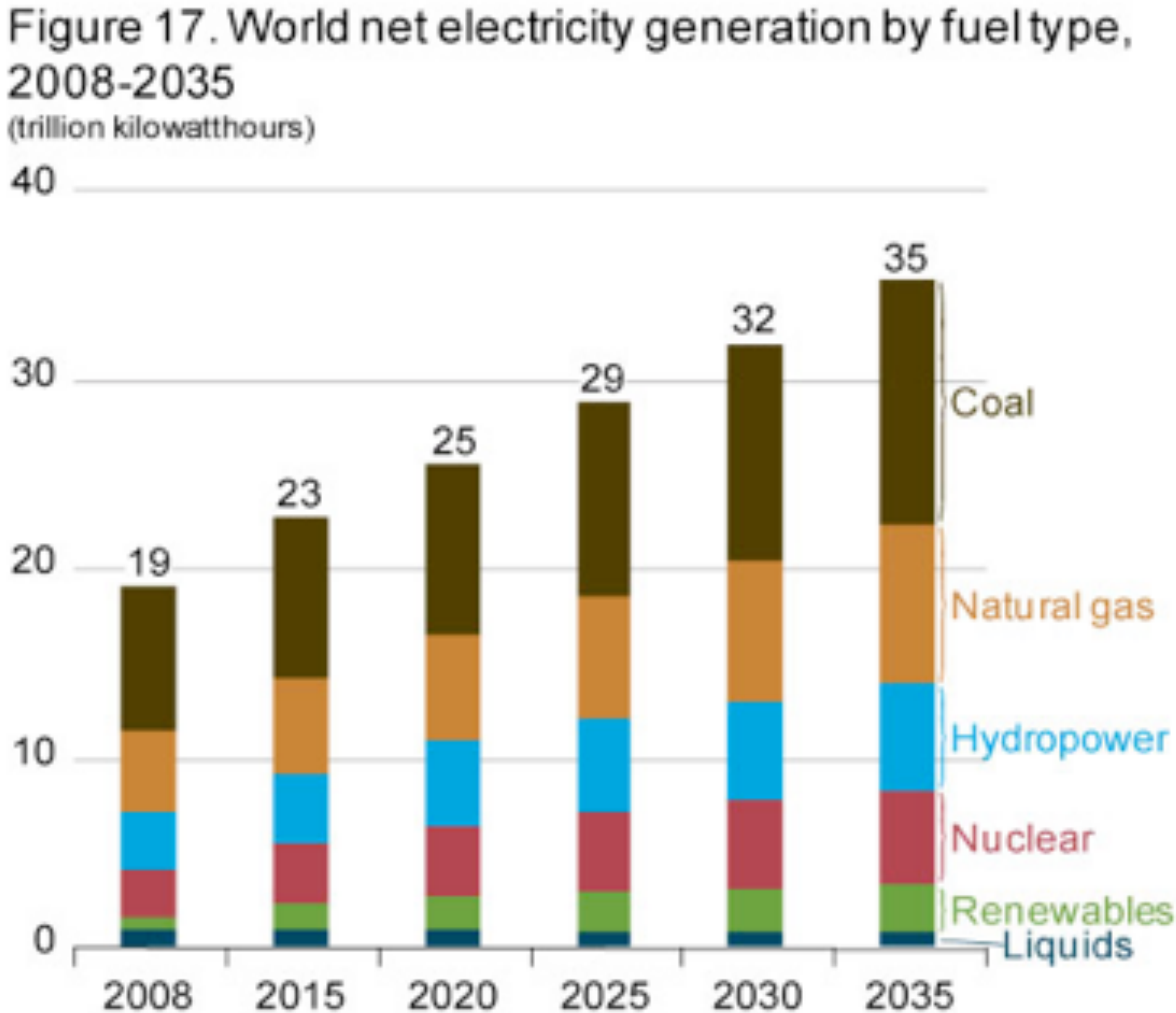


* IEA estimate
Source: IEA databases and analysis.

World electricity usage – by region



World electricity usage – by fuel type



What about Coal & Natural Gas?

Energy density and carbon impact

See also: Energy value of coal

The energy density of coal, i.e. its heating value, is roughly 24 megajoules per kilogram[79] (approximately 6.7 kilowatt-hours per kg). For a coal power plant with a 40% efficiency, it takes an estimated 325 kg (720 lb) of coal to power a 100 W lightbulb for one year.[80]

As of 2006, the average efficiency of electricity-generating power stations was 31%; in 2002, coal represented about 23% of total global energy supply, an equivalent of 3.4 billion tonnes of coal, of which 2.8 billion tonnes were used for electricity generation.[81]

The US Energy Information Agency's 1999 report on CO_2 emissions for energy generation quotes an emission factor of 0.963 kg CO_2/kWh for coal power, compared to 0.881 kg CO_2/kWh (oil), or 0.569 kg CO_2/kWh (natural gas).[82]


Now on to a new day…