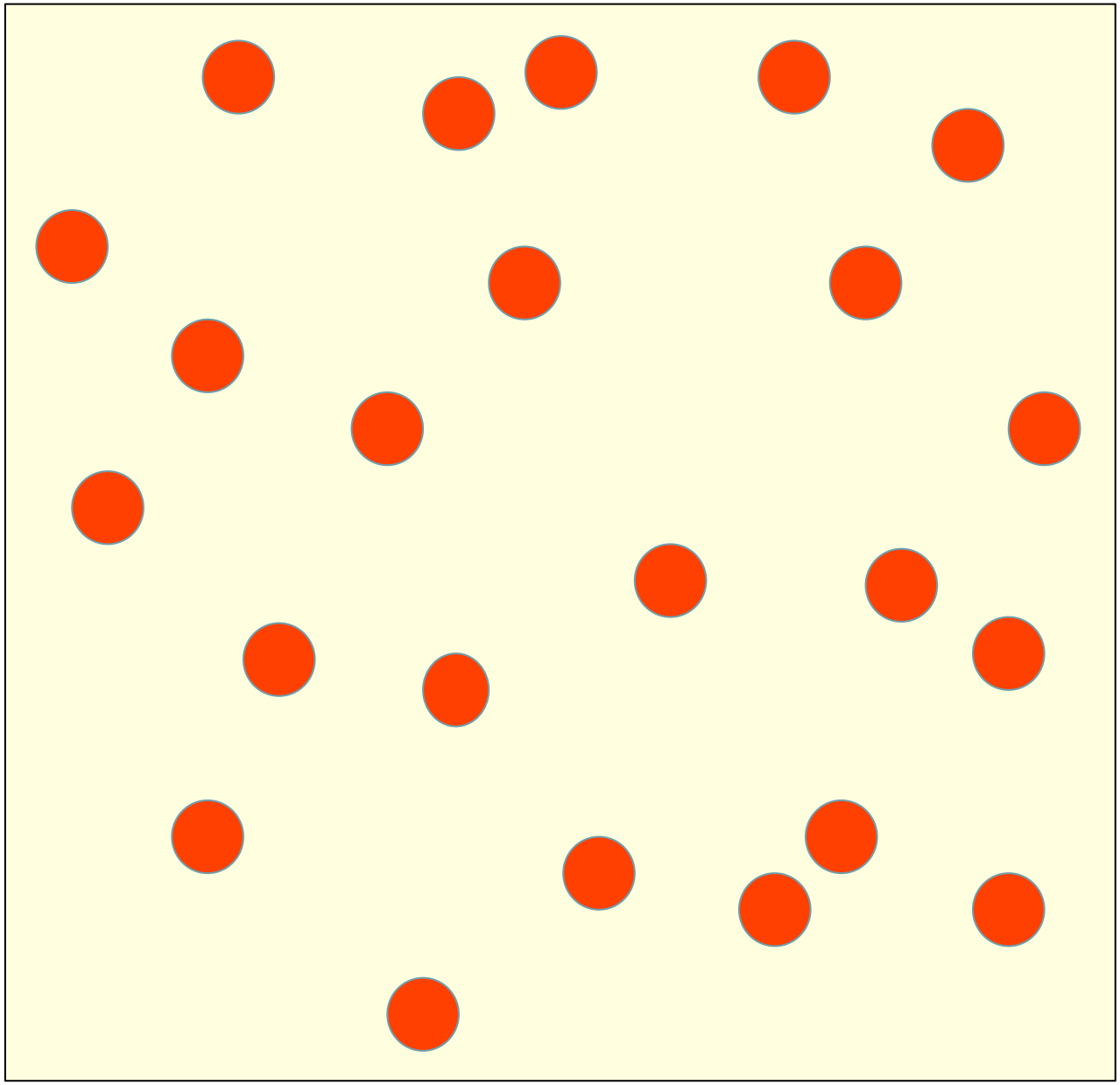
Lecture 6
Chain Reactions & Criticality

First a quick reprise of reaction cross sections

Measuring the diameter of a bowling ball without a ruler

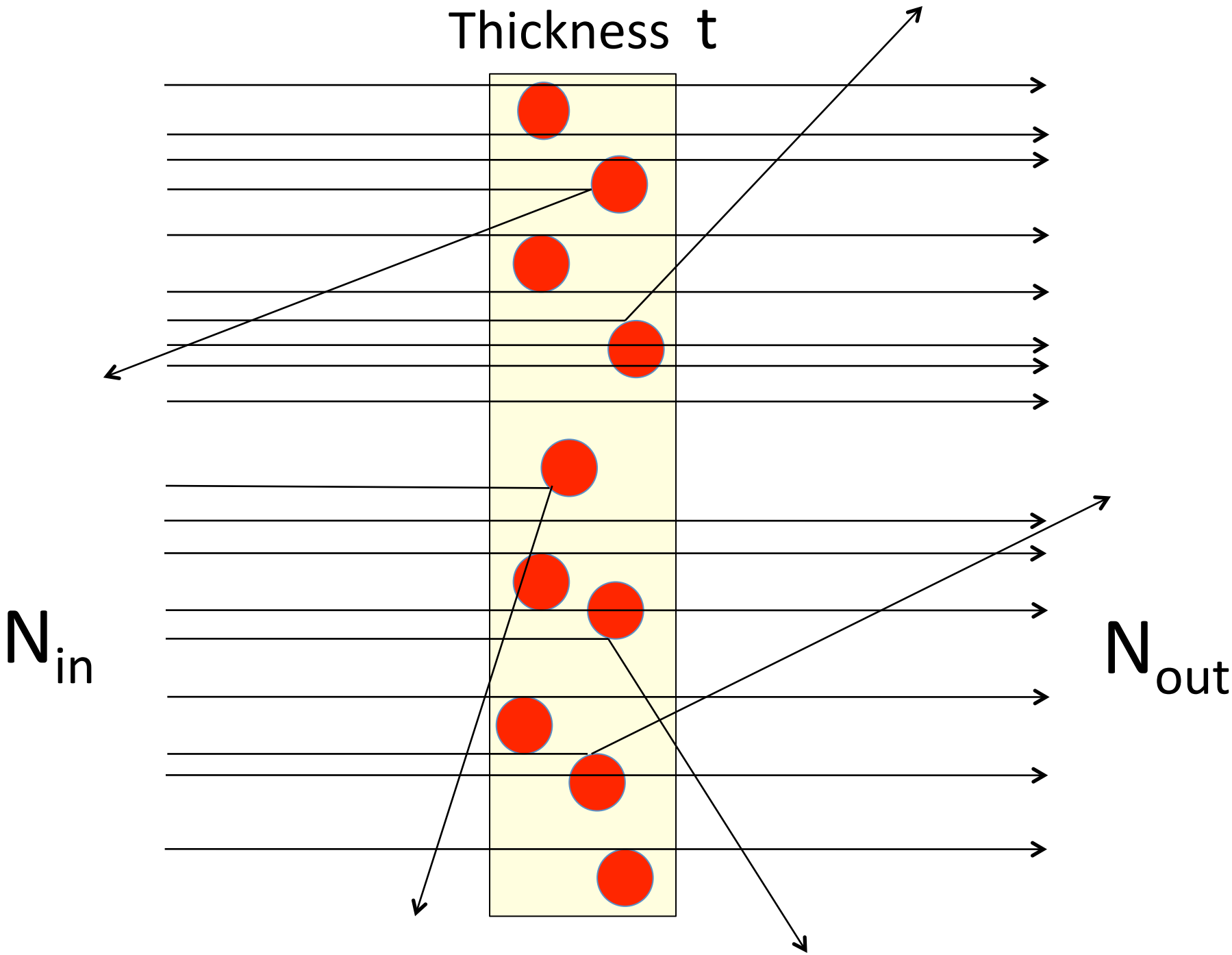


Bale of hay, containing bowling balls of unknown cross-sectional area σ .

The density of bowling balls is given by **n** [m^{-3}] .

This is the front view, but you really can't see the bowling balls! They're hidden inside.

Side view – now let's blast away with a shotgun



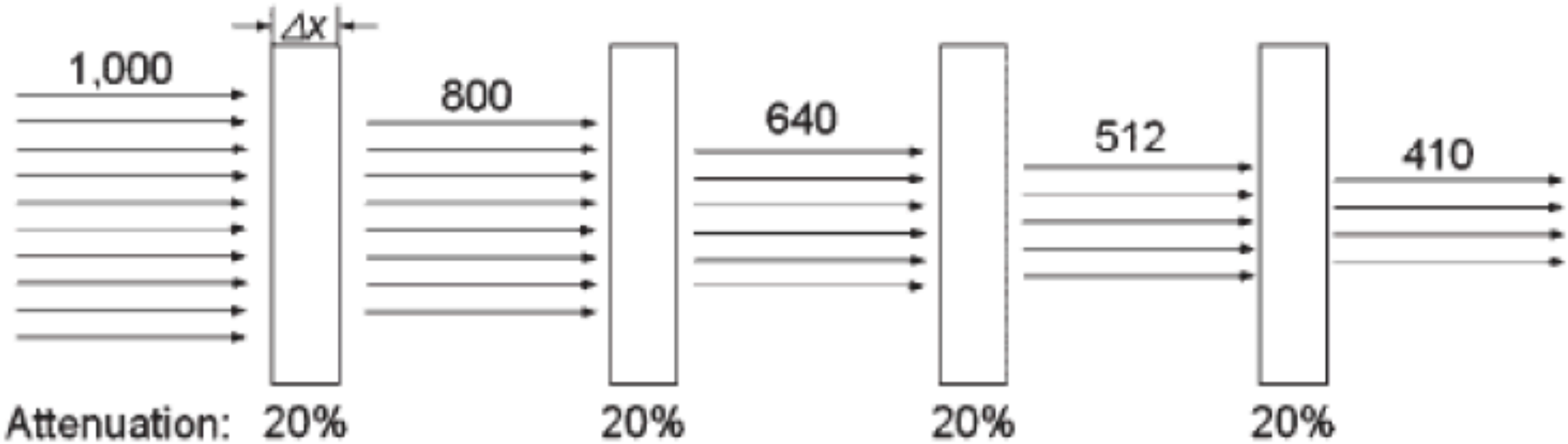
Definition of a cross-section, $\sigma[cm^2]$

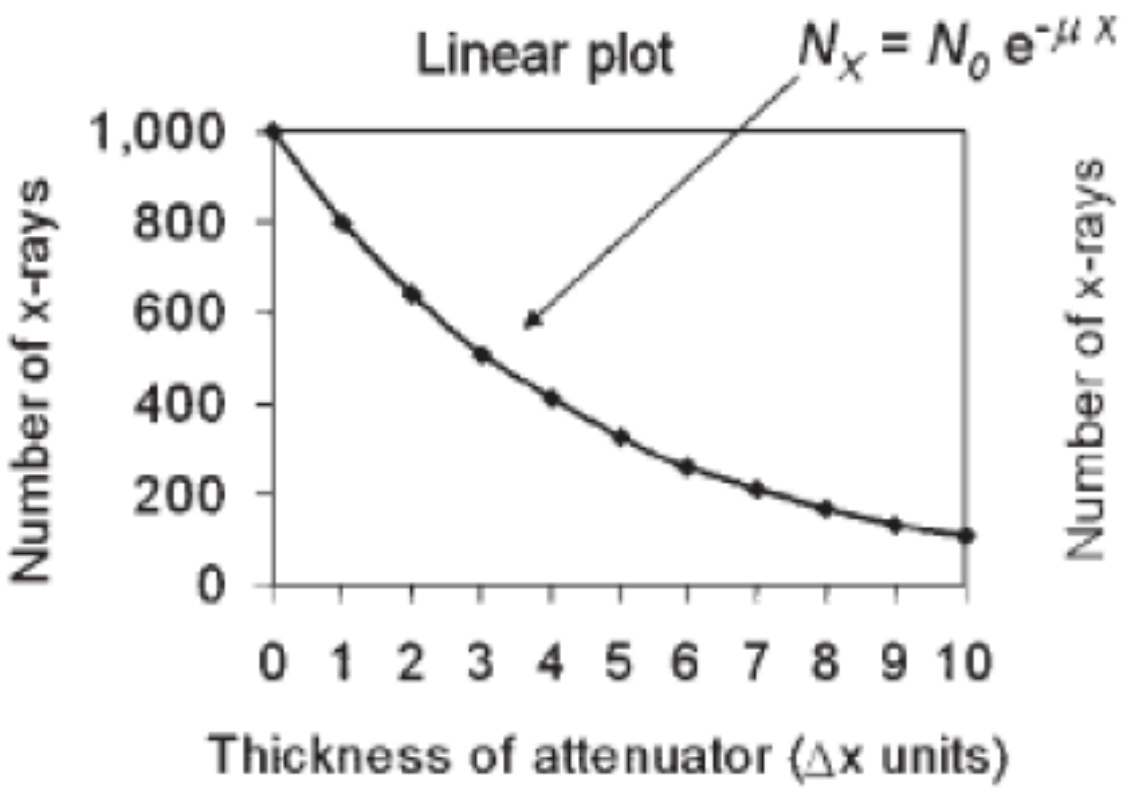
$$N_{scatter} = N_{in} \underbrace{n[cm^{-3}]\ t\ [cm]}_{\rho_{\#}\ [cm^{-2}]}\ \sigma[cm^2]$$

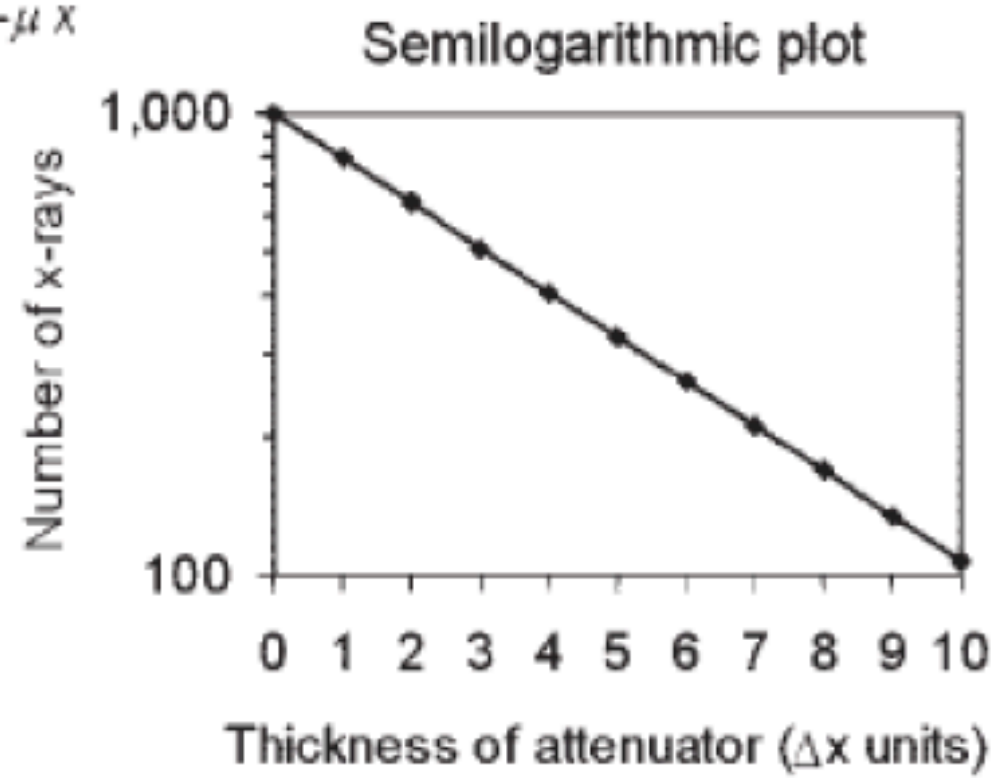
The areal density, i.e. the *number* of scattering centers per square centimeter

The basis for measuring a cross section is that the probability of scattering is the fraction of area occluded by the target nuclei

Attenuation curves & concept of Mean Free Path (m.f.p.)







Attenuation curves & concept of Mean Free Path (m.f.p.)

- The Mean Free Path (m.f.p.) is that distance for which the unattenuated (unscattered) beam has been reduced to 1/e = 0.3679 of its original value.

- Examining the expression in the previous viewgraph, the mean-free path, often written λ , is thus given by $\lambda = 1/\mu$. This begs the question, however, what is μ ?

- Quite plainly then:

Depends on density & material

$$\mu\ [\mathrm{cm}^{-1}] = \rho_{\#}\ [\mathrm{cm}^{-3}]\ \sigma\ [\mathrm{cm}^{2}]$$

Depends on underlying nuclear physics

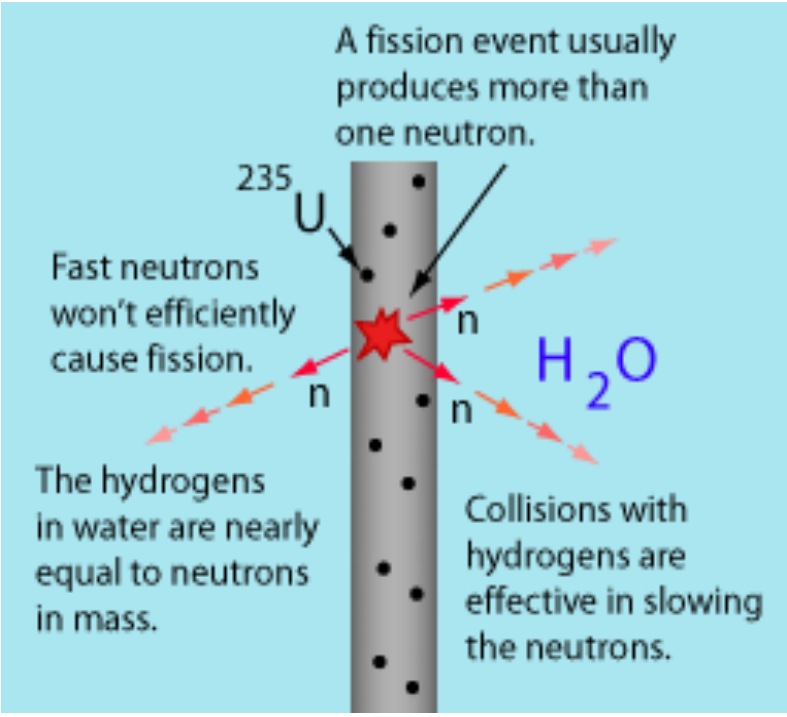
What are the two isotopes from which one can most readily make either a nuclear reactor or a weapon?

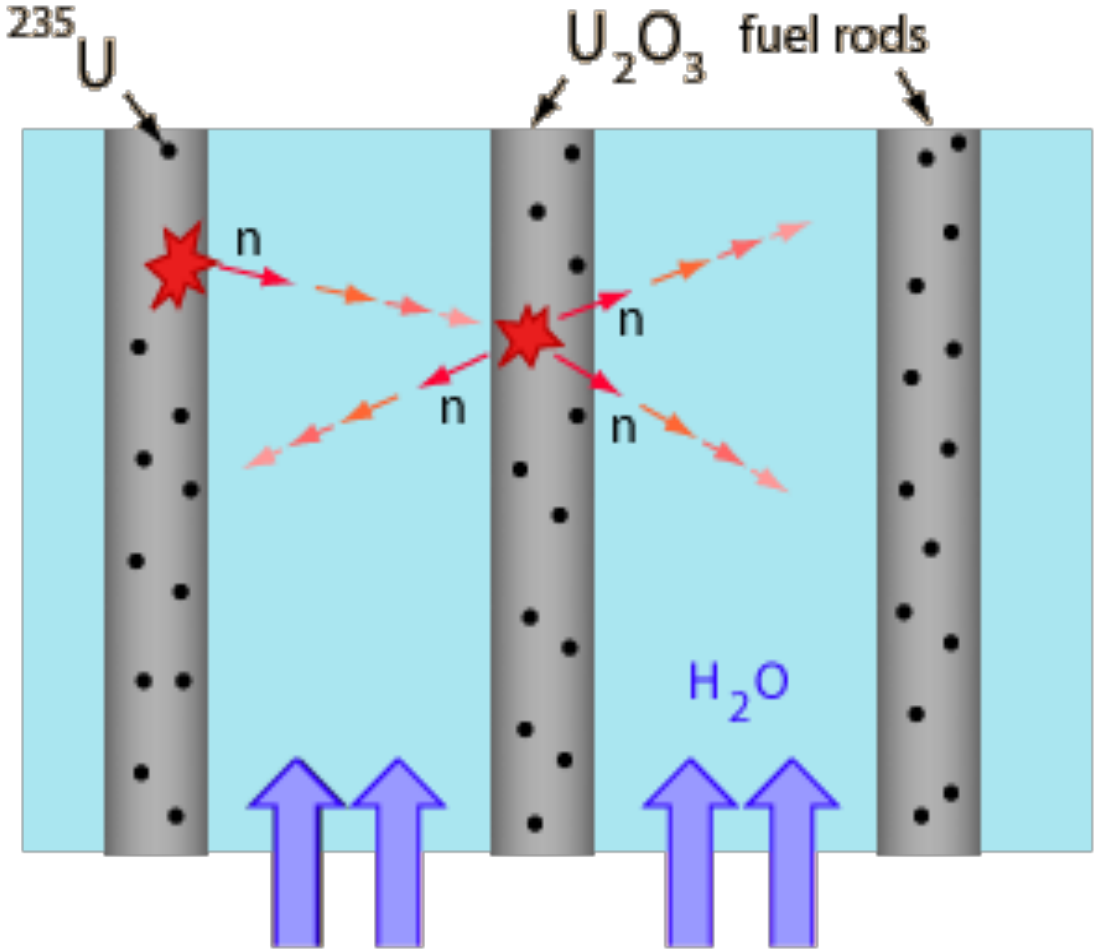
- **^{235}U**
  - Minority component of natural U (0.7%)
  - Four separation methods: electromagnetic, gaseous diffusion, centrifuge, laser isotope
- **^{239}Pu**
  - Produced as a by-product in ^{235}U reactors
  - Then can be easily chemically separated
  - $$^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{239}_{92}\mathrm{U} \xrightarrow[23.5\ \mathrm{min}]{\beta^-} {}^{239}_{93}\mathrm{Np} \xrightarrow[2.3565\ \mathrm{d}]{\beta^-} {}^{239}_{94}\mathrm{Pu}$$

Both fission upon capture of *slow* neutrons (thermal)

Now a review of reactor physics basics

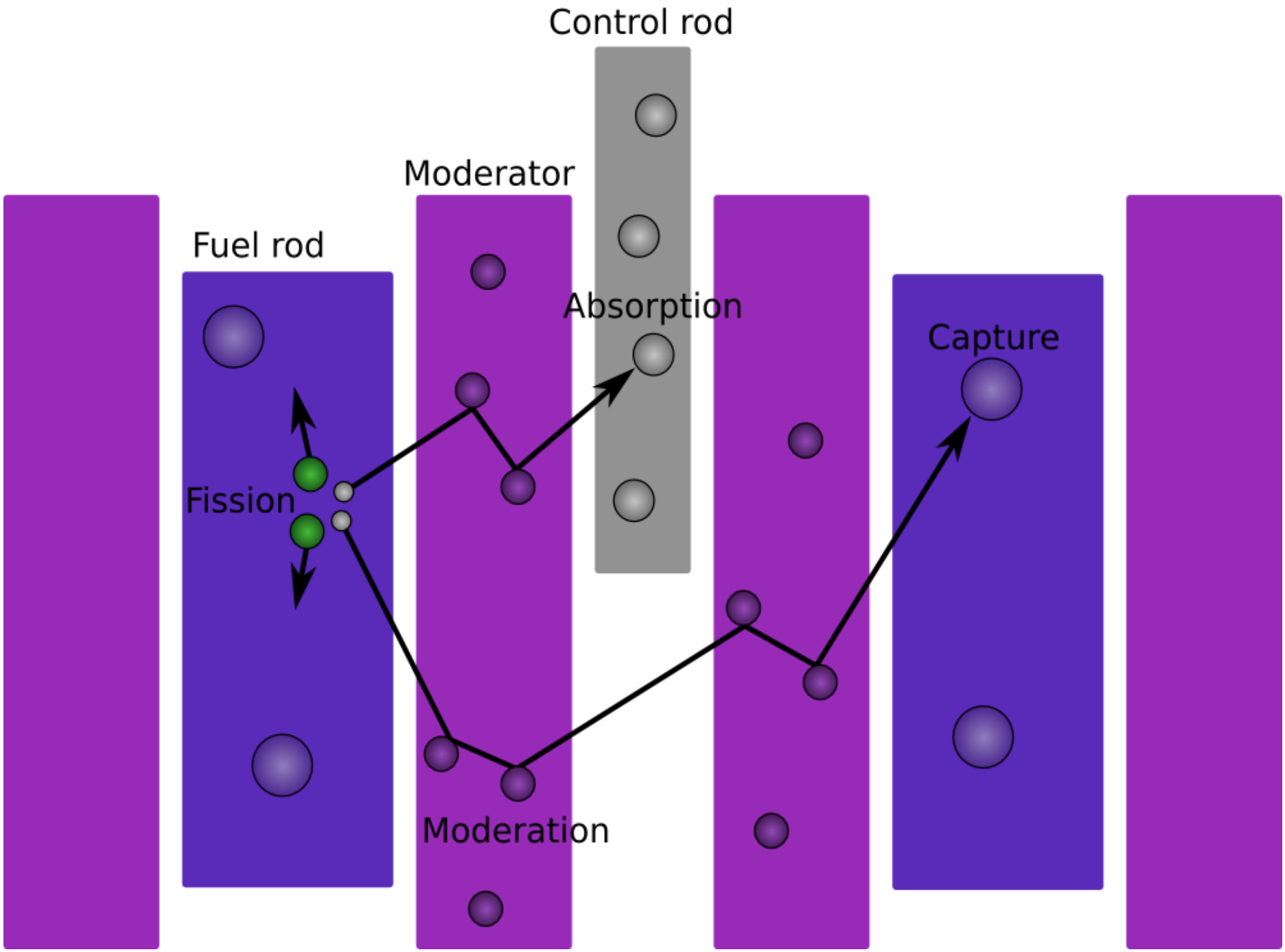
The general concept of a reactor…





Water as coolant and moderator flows between fuel rods.

A little better picture



Neutron Multiplication Factor

- $k < 1$ Subcritical
- $k = 1$ Critical
- $k > 1$ Supercritical
- $k >> 1$ Bomb

The life of each neutron in a reactor is like "Indiana Jones and the Temple of Doom"

4-Factor Formula – fuel of *infinite* extent

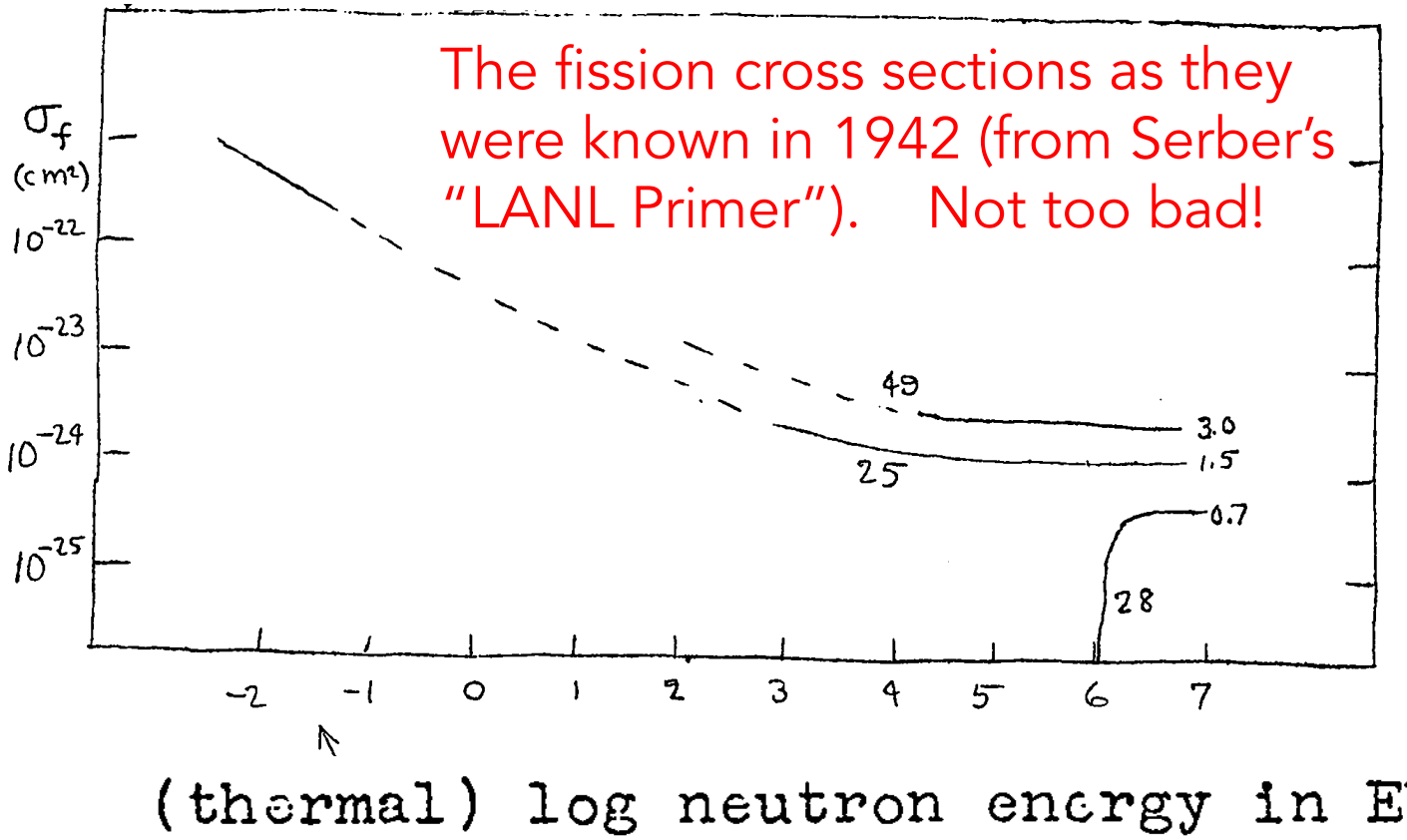
$$k_\infty = \eta\, \varepsilon\, p\, f$$

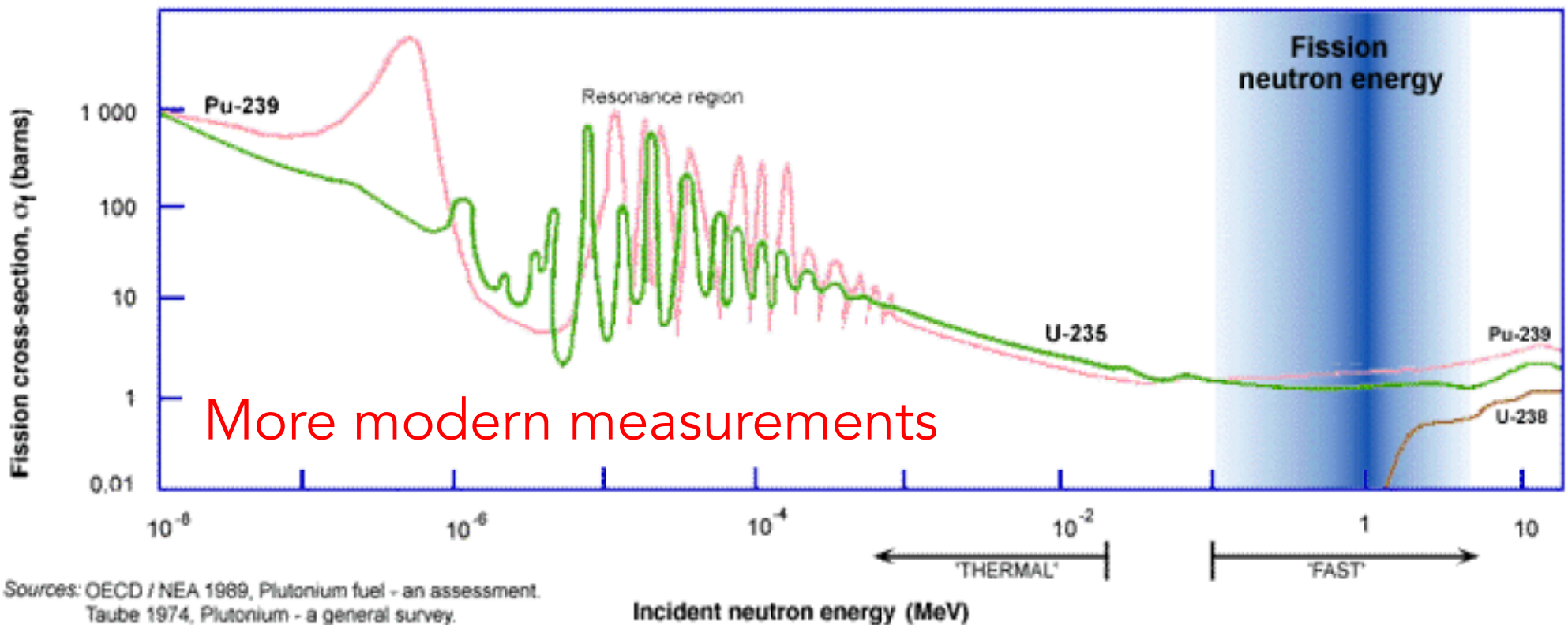
η = Number of neutrons emitted per neutron captured in the fuel

ε = Fraction of fast neutrons that induce fission

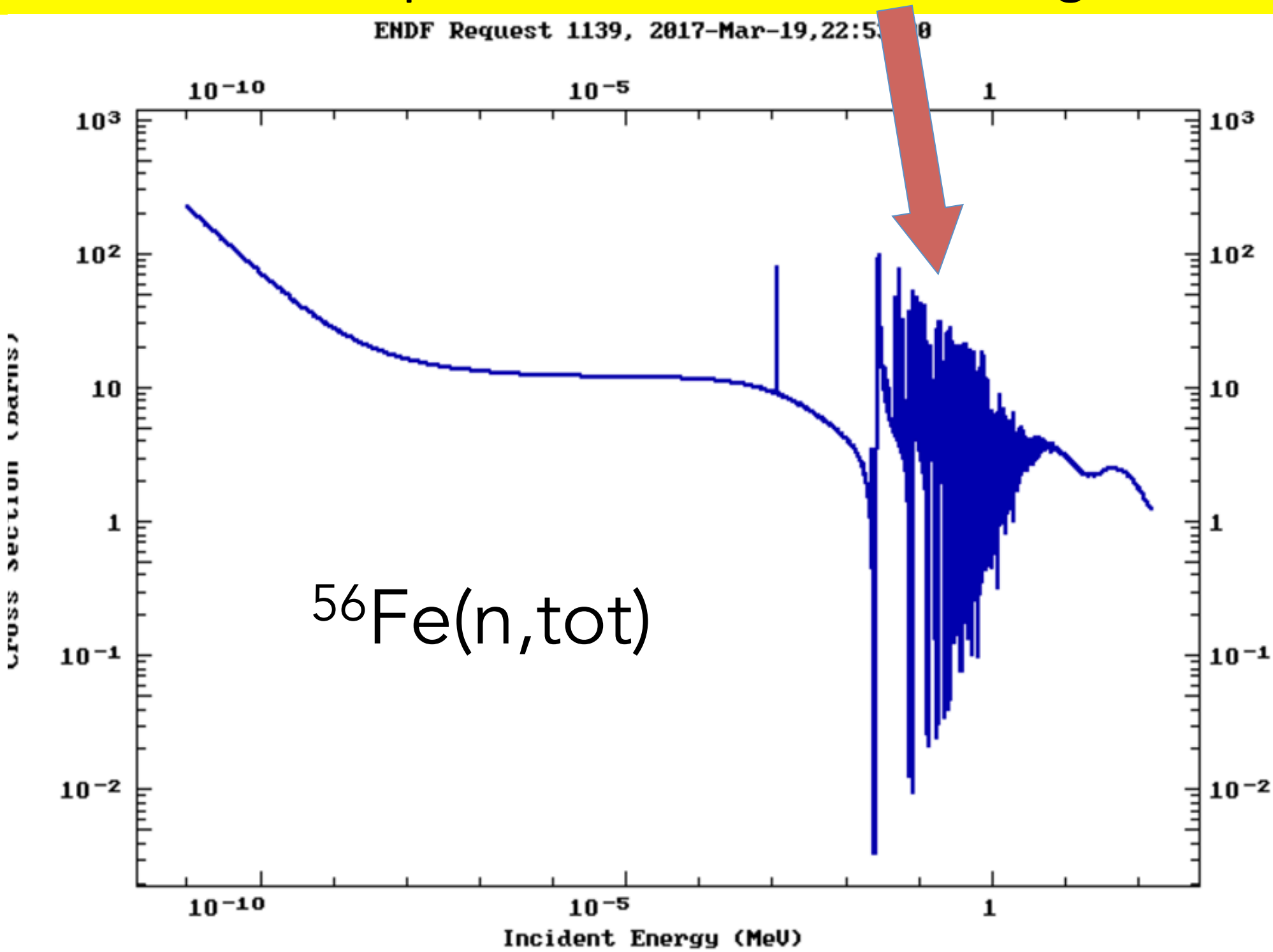
p = Fraction of neutrons surviving resonance region

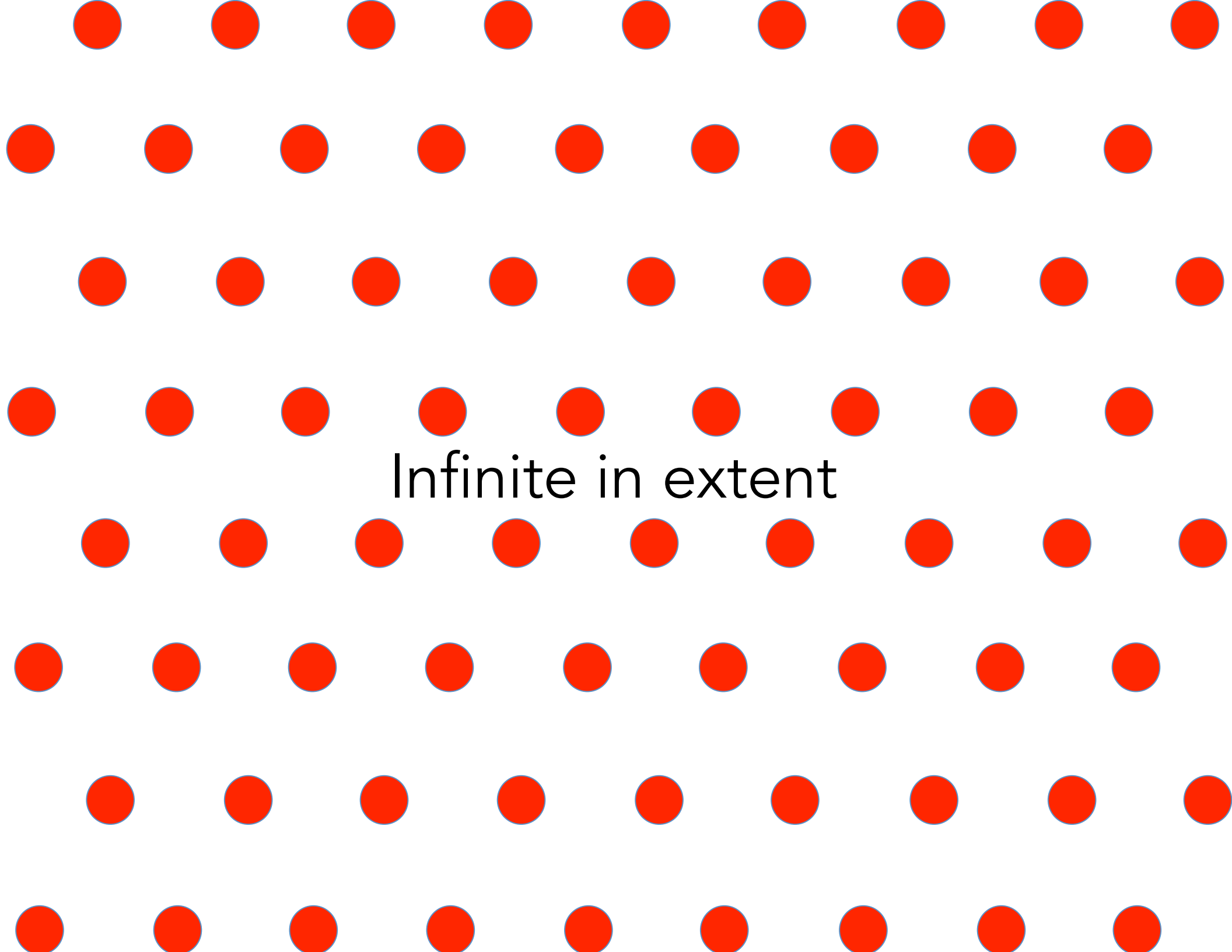
f = Fraction of thermal neutrons captured by fuel


The fission cross sections as they were known in 1942 (from Serber's "LANL Primer"). Not too bad!
σ_f (cm²)
10^{-22}
10^{-23}
10^{-24}
10^{-25}
49
3.0
25
1.5
0.7
28
-2
-1
0
1
2
3
4
5
6
7
(thermal) log neutron energy in EV.

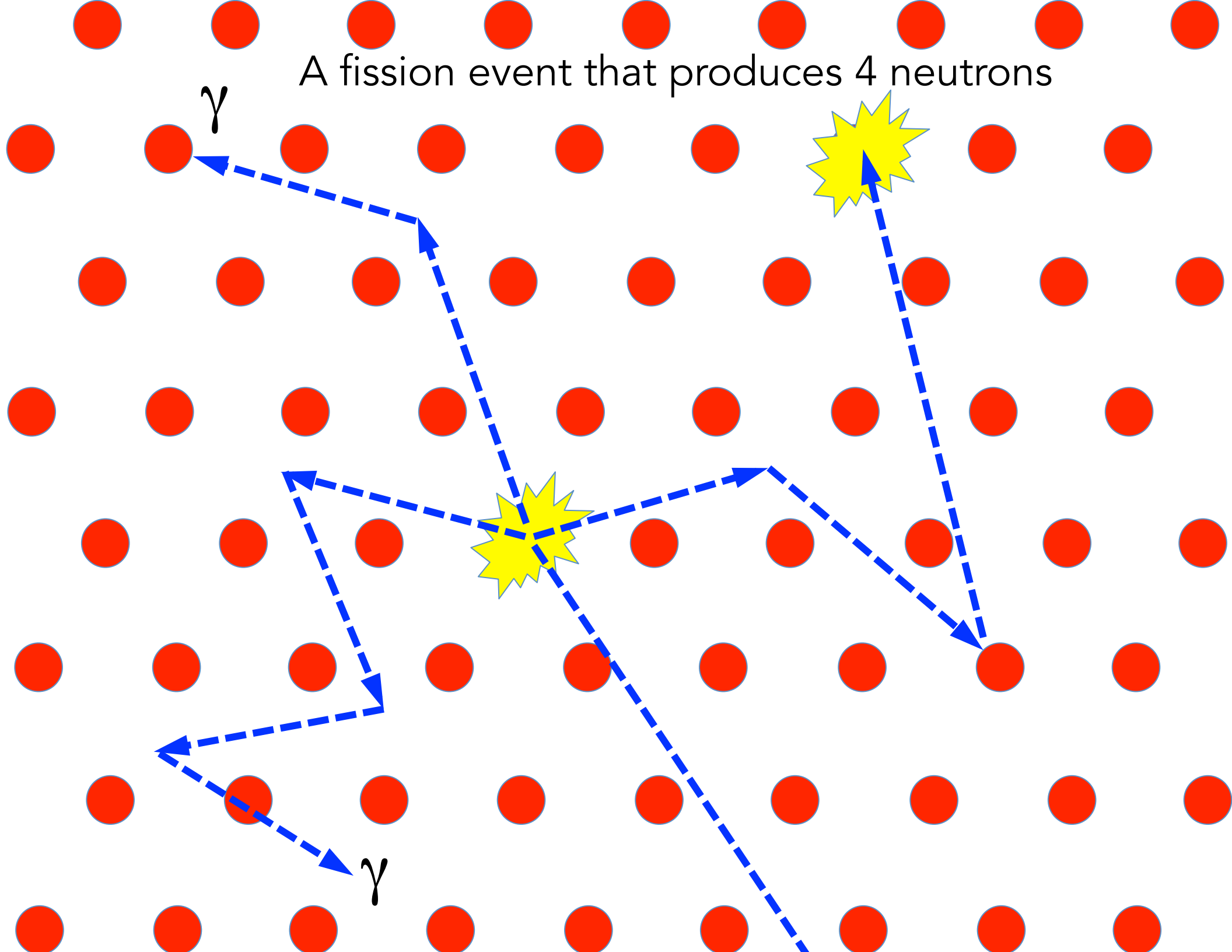

More modern measurements
Fission cross-section, σ_f (barns)
1 000
100
10
1
0.01
Pu-239
Resonance region
Fission neutron energy
U-235
Pu-239
U-238
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10
'THERMAL'
'FAST'
Incident neutron energy (MeV)
Sources: OECD / NEA 1989, Plutonium fuel - an assessment.
Taube 1974, Plutonium - a general survey.

Neutron capture in the Resonance Region

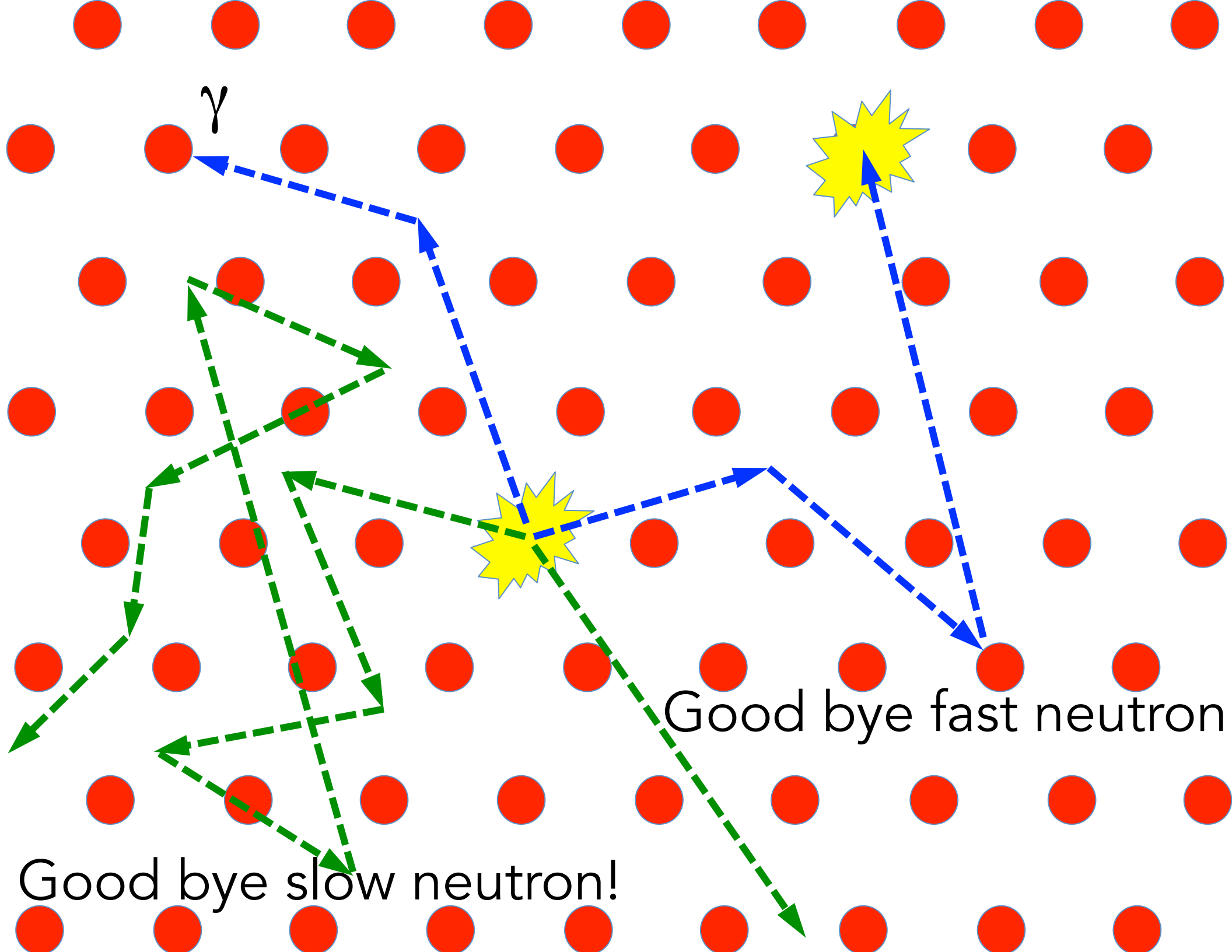



Infinite in extent

A fission event that produces 4 neutrons



Finite in extent – Just what you see here


γ
Good bye fast neutron
Good bye slow neutron!

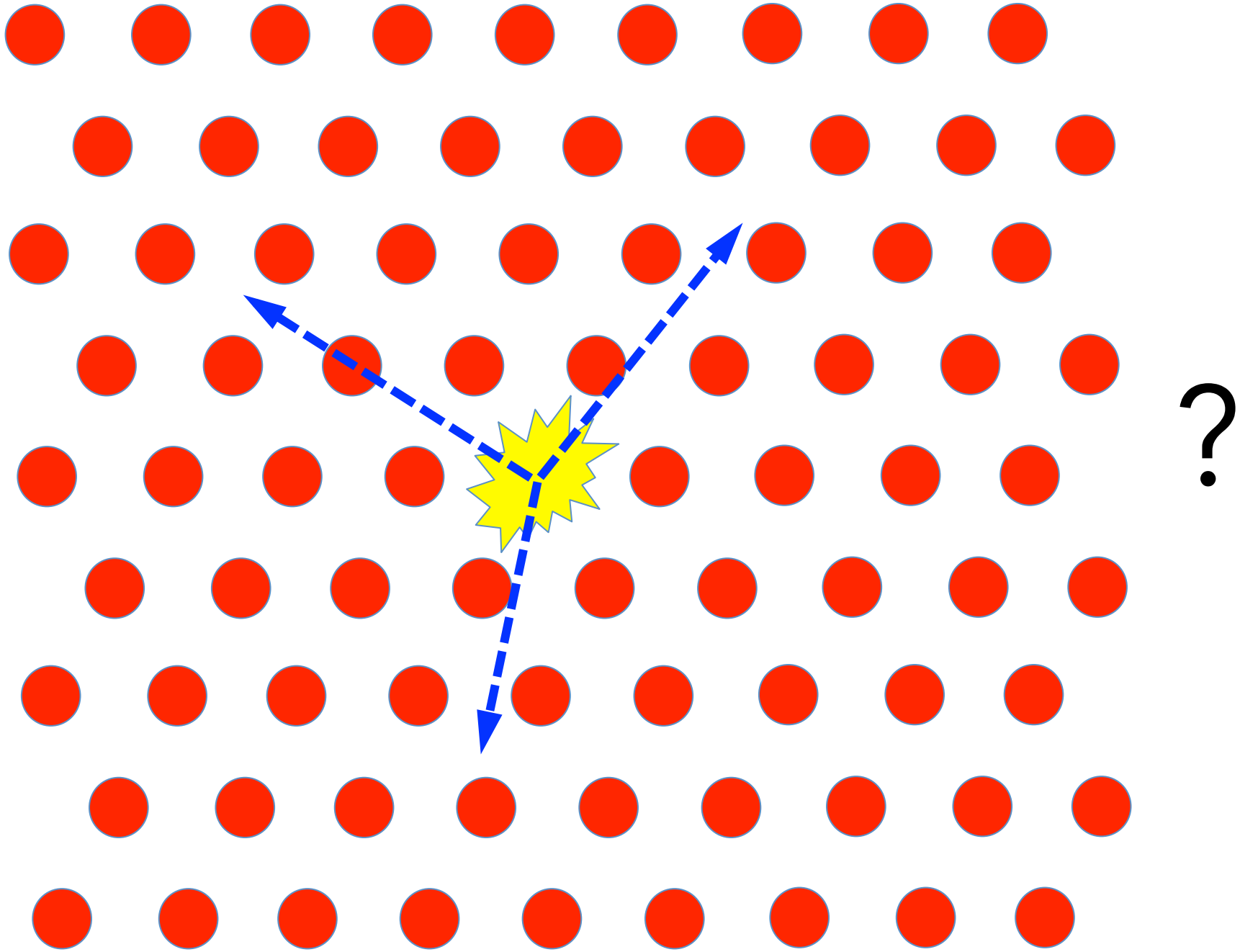
6-Factor Formula – Fuel of *finite* extent

$$k = \eta \, \varepsilon \, p \, f \, L_f \, L_t$$

L_f = Probability *fast* neutron does not escape the assembly

L_f = Probability *slow* neutron does not escape the assembly

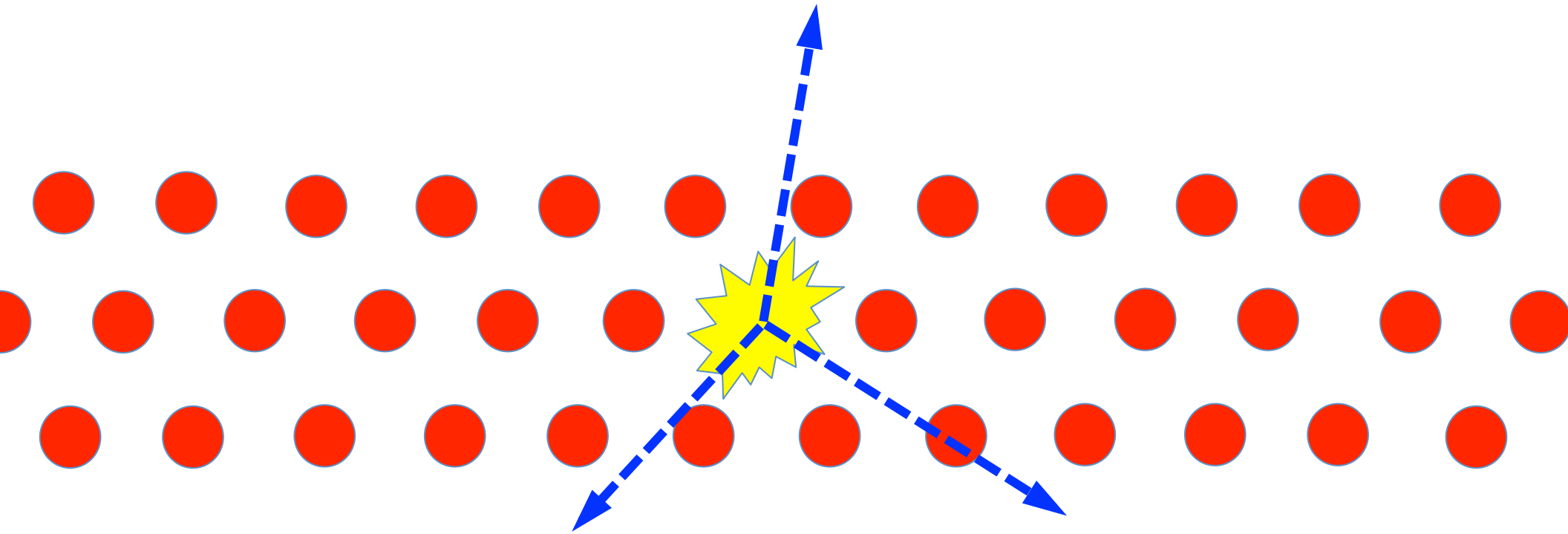
So total MASS is important



Ahah!

Not only is MASS important,

But DENSITY is important



Same total mass, but now extended in both directions

So what's important now?

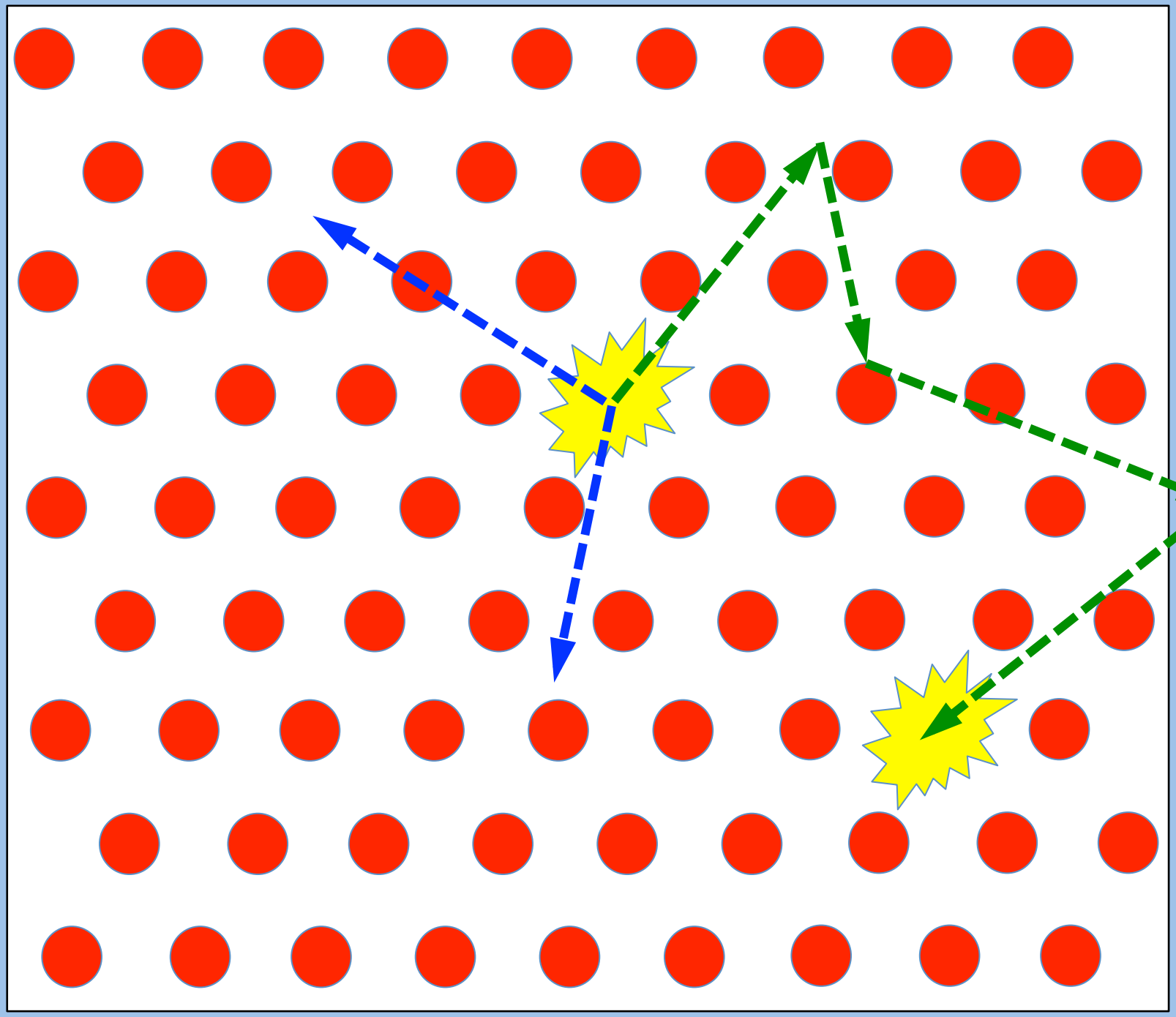
MASS,

DENSITY,

SHAPE

But hold on, one more coming …

Neutron Reflector (sometimes dual purpose as ‘tamper’)



At long last:

MASS,

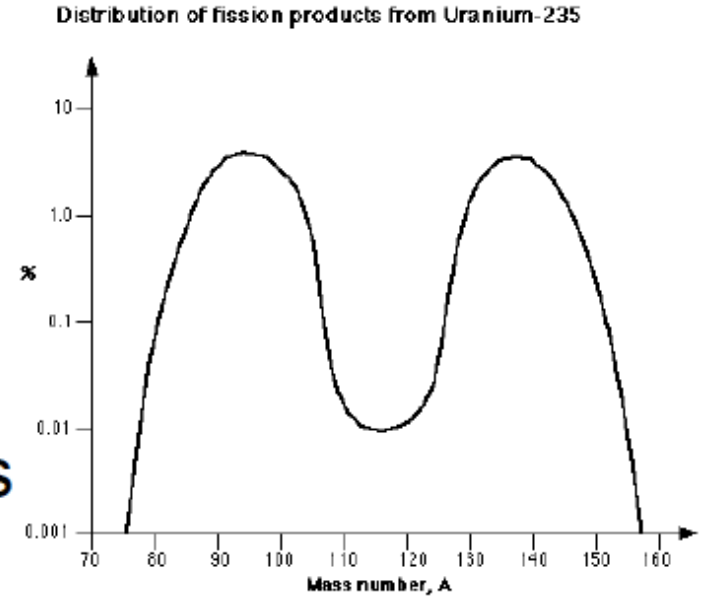
DENSITY,

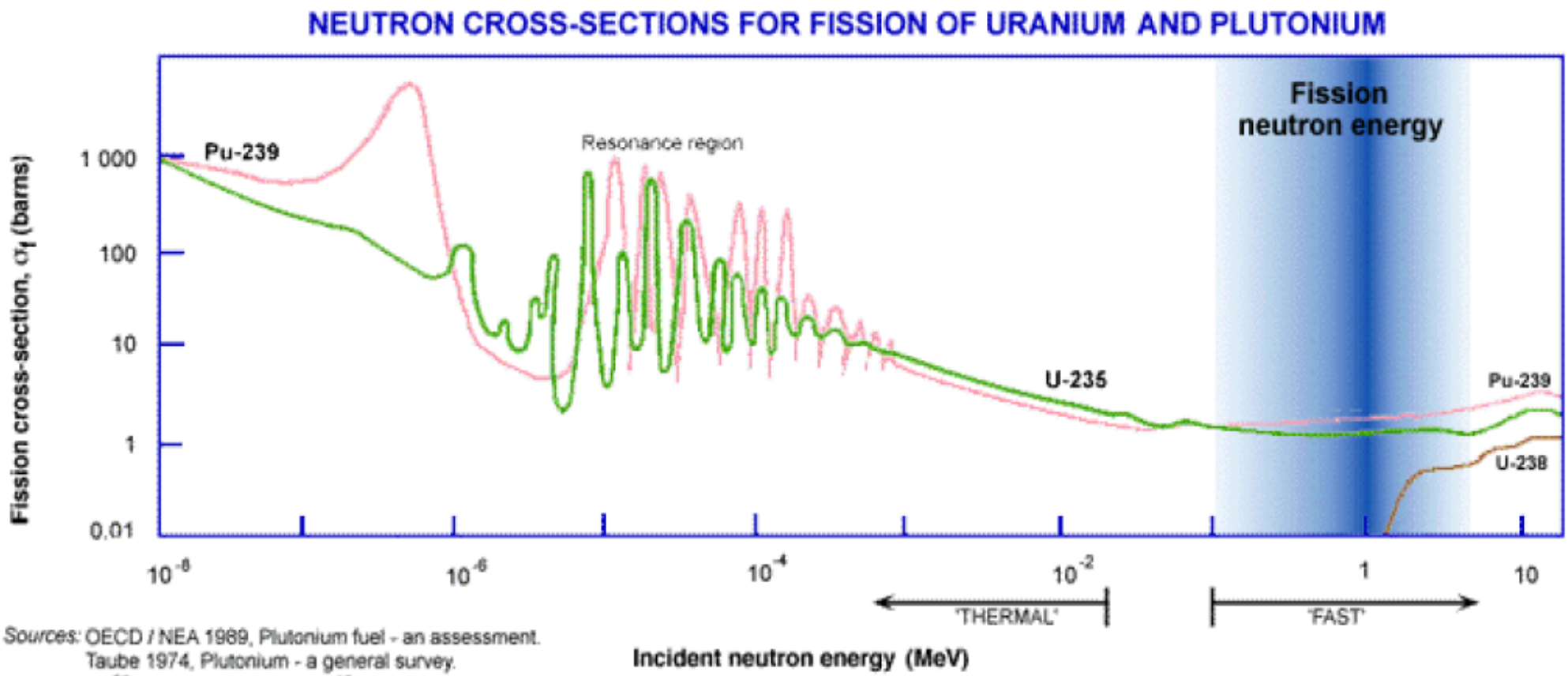
SHAPE,

NEUTRON REFLECTION

Fission cross section

Experimental results from fission studies





Sources: OECD / NEA 1989, Plutonium fuel - an assessment.
Taube 1974, Plutonium - a general survey.
1 barn = 10^{-28} m2, 1 MeV = 1.6 x 10^{-13} J