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Independent and Cumulative Fission Yield Covariance Matrices for 61 Compound Systems

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Introduction

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Motivation

• The 1994 fission yield evaluation by England and Rider does not include information on covariances between fission yields. [1]

• Covariances between fission yields affect a number of important applications:
  – Forensics and safeguards calculations
  – Reactor antineutrino rates
  – Reactor inventory, decay heat, and poisoning

Previous Work

- **Pigni et al. – 2013**
  - Variance estimation with Wahl systematics
- **Schmidt – 2013**
  - Parameters perturbation in the GEF code
- **Leray et al. – 2017**
  - Parameters perturbation in the GEF code
- **Kawano and Chadwick – 2013**
  - Bayesian method for $^{239}$Pu FPY

- Work by Pigni, Schmidt, and Kawano presented in WPEC Subgroup 37
- Work by Pigni, Schmidt, and Leray relies on an underlying model of fission and parameter uncertainties.
- Results of these work are not readily accessible due in part to ENDF format limitations.
Motivation

- The goal of this work is to generate a set of covariance matrices for the fissioning systems of the England and Rider evaluation with as little fission model bias/uncertainty as possible.
- This method seeks to use simple conservation rules in order to constrain a sample space for Monte-Carlo bootstrapping.
- The resulting covariance matrix will predominantly reflect the evaluated uncertainties in the independent fission yields.
- Once these matrices are generated, making them available online will be a priority.
Monte-Carlo Uncertainty Propagation

- Given a dataset with characterized uncertainty; one builds a new series of datasets by resampling the original one.
  - This can be used to assess uncertainties and covariance in an output calculation by varying the input data.
  - It could also be used to assess covariances between the values in the original dataset.

- This can be applied to generate covariance matrices for FYs:
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- This can be applied to generate covariance matrices for FYs:

  However, resampling fission yields like this – independently of each other – will yield **no correlation/covariance**.
Conserved Quantities

- In order to obtain correlation, conserved quantities can be enforced upon a set of resampled fission yields [1]:

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**Charge Parity:**
\[ \sum_{i} Y_i(Z_1, A_i) = \sum_{i} Y_i(Z_{CN} - Z_1, A_i) \]

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\sum_i A_i Y_i = A_{CN} - \bar{\nu}
\]

Charge Parity:

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\sum_i Y_i (Z_1, A_i) = \sum_i Y_i (Z_{CN} - Z_1, A_i)
\]

Mass Symmetry:

\[
\sum_{A_i > \frac{A_{CN} - \bar{\nu}}{2}} Y_i (A_i) = 1
\]

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\[
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\[
\sum_i A_i Y_i = A_{CN} - \bar{v}
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Charge Parity:

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\sum_i Y_i (A_i) = 1
\]

Mass Symmetry:

This relationship is only approximately conserved. It is debatable whether it is a valid condition. Nevertheless, it is exploited in order to help conserve the other 5 relationships.

FY Covariance Matrix Generation

- The way in which a set of fission yields are resampled can be structured to conserve these relationships:
  - 1) Randomly selected the “light” or “heavy” side of the fission product spectrum to resample.
  - 2) Randomly select (weighted by uncertainty) a product in each $A$ chain, resample its yield about its evaluated uncertainty.
  - 3) Scale all other yields in that $A$ chain by the same percent change.
• The way in which a set of fission yields are resampled can be structured to conserve these relationships:

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Step 3 is allowed if the $Z$ distribution for a given $A$ is Gaussian, which empirical data and the E&R evaluation supports [1].
FY Covariance Matrix Generation

- **4)** Normalize the resampled yields such that they sum to 1.
- **5)** Generate the fission yields on the complementary side of the fission product spectrum using the neutron multiplicity of the compound system.

\[
Y_{\text{frac}}(Z_{\text{CN}} - Z, A_{\text{CN}} - A - \nu) = P(\nu) \cdot Y(Z, A)
\]

\[
Y(Z_{\text{CN}} - Z, A_i) = \sum_{\nu} Y_{\text{frac}}(Z_{\text{CN}} - Z, A_i)
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By Step 5 we’ve ensured all of the conservation rules are met.
6) Repeat steps 1-5) $N$ times. Select $N$ such that statistical noise is minimized.

7) Calculate the resulting correlation matrix from the $N$ trials.

Correlation matrix for independent fission yields of $^{235}\text{U}$ fast fission.
• The England and Rider evaluation does not make any mention of the neutron multiplicity distribution used for their evaluations.

• Thus we are left to assume a neutron multiplicity distribution that sufficiently matches the England and Rider evaluation.

Neutron Multiplicity for fast neutron induced fission of $^{235}\text{U}$ according to J.P. Lestone in LA-UR-05-0288.
FY Covariance Matrix Generation

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However, we know $P(\nu)$ has dependence on $A$ and $Z$ and energy, etc.

Neutron Multiplicity for fast neutron induced fission of $^{235}\text{U}$ according to J.P. Lestone in LA-UR-05-0288.
E&R Consistent $P(\nu, A)$ Data

- $P(\nu, A)$ can be fitted to the England and Rider evaluation in order to obtain the best degree of consistency.
- A truncated Gaussian is used to fit the shape of the $P(\nu)$ distribution for each $A$ chain.
- Select $P(\nu, A)$ that minimizes $\chi^2$ between evaluated yields and “recalculated yields”, $Y'(Z, A)$

$$Y'(Z, A) = \sum_{\nu} P(\nu, A) Y(Z_{CN} - Z, A_{CN} - A - \nu)$$
E&R Consistent $P(\nu, A)$ Data

**Example:**
Reproduction of evaluated yields to obtain $P(\nu, 135)$ for fast fission of $^{235}\text{U}$. 
E&R Consistent $P(\nu, A)$ Data

Example: Resampled yields for $^{132}\text{Te}$:

Using simple $P(\nu)$ data

Using E&R consistent $P(\nu, A)$ data
• **Example:** $^{135}$Te

• Presented is the covariance between independent yields as function of $Z$ and $A$ and that of $^{135}$Te.

• The evaluated yield for $^{135}$Te is $2.47 \pm 0.57\%$
Expected Behavior

- **Features:**
- $^{135}\text{Te}$ is positively correlated with itself.
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  - $^{135}\text{Te}$ is positively correlated with itself.
  - Products along the $A$ chain have positive correlation.
    - This positive correlation is reflected along a complementary $A = 99$ chain.
  - Products along $A$ chains that do not have complementary $Z$ have negative correlation.
Conclusions

• A model-agnostic method for independent fission yield covariance matrix generation is being developed.

• This method has been successfully applied to all 61 compound systems in the England and Rider evaluation.

• The results demonstrate expected behavior and trends.

• Final results serve as an interim solution for independent fission yield covariance matrices until a new evaluation is completed.
  – The results are publicly available at nucleardata.berkeley.edu/FYCOM
  – A peer-reviewed publication on this method has been submitted to journal.
NSSC Experience


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