Nuclear Science and Security Consortium
Virtual Scholar Showcase 2020

Independent and Cumulative Fission Yield Covariance Matrices for 61 Compound Systems

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NUCLEAR SCIENCE and SECURITY CONSORTIUM

Introduction

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Bruce D. Pierson

Motivation

- **The 1994 fission yield evaluation by England and Rider does not include information on covariances between fission yields. [1]**
- **Covariances between fission yields affect a number of important applications:**
  - Forensics and safeguards calculations
  - Reactor antineutrino rates
  - Reactor inventory, decay heat, and poisoning

[1] – Evaluation and Compilation of Fission Product Yields – T.R. England and B.F. Rider (1994)

Previous Work

- **Pigni et al. – 2013**
  - Variance estimation with Wahl systematics
- **Schmidt – 2013**
  - Parameters perturbation in the GEF code
- **Leray et al. – 2017**
  - Parameters perturbation in the GEF code
- **Kawano and Chadwick – 2013**
  - Bayesian method for ^{239}Pu FPY

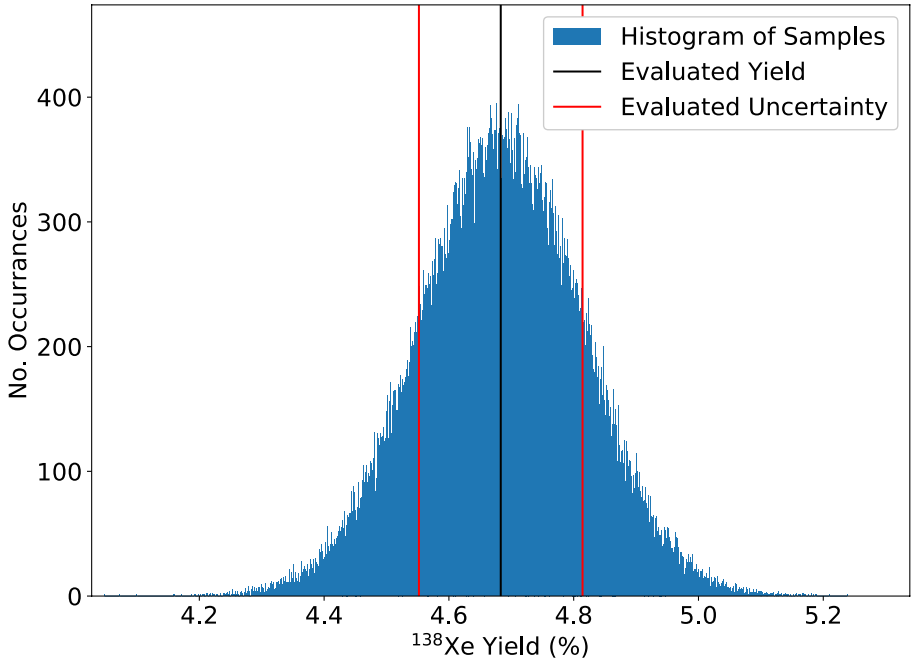
- Work by Pigni, Schmidt, and Kawano presented in WPEC Subgroup 37
- Work by Pigni, Schmidt, and Leray relies on an underlying model of fission and parameter uncertainties.
- Results of these work are not readily accessible due in part to ENDF format limitations.

Motivation

- The goal of this work is to generate a set of covariance matrices for the fissioning systems of the England and Rider evaluation with as little fission model bias/uncertainty as possible.
- This method seeks to use simple conservation rules in order to constrain a sample space for Monte-Carlo bootstrapping.
- The resulting covariance matrix will predominantly reflect the evaluated uncertainties in the independent fission yields.
- Once these matrices are generated, making them available online will be a priority.

Monte-Carlo Uncertainty Propagation

- **Given a dataset with characterized uncertainty; one builds a new series of datasets by resampling the original one.**
  - This can be used to assess uncertainties and covariance in an output calculation by varying the input data.
  - It could also be used to assess covariances between the values in the original dataset.
- **This can be applied to generate covariance matrices for FYs:**



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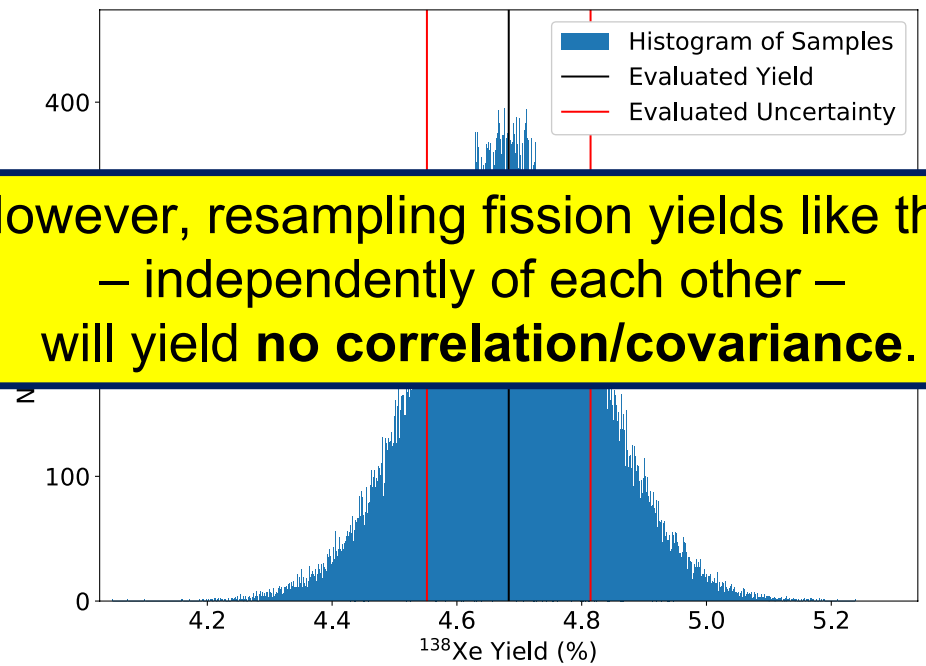

However, resampling fission yields like this
– independently of each other –
will yield **no correlation/covariance**.

Conserved Quantities

- **In order to obtain correlation, conserved quantities can be enforced upon a set of resampled fission yields [1]:**

Total Yield:

$$\sum_i Y_i = 2$$

[1] - Generation of Fission Yield Covariances to Correct Discrepancies in the JEFF Fission Yield Library – L. Fiorito et al. (2015) - https://www.oecd-nea.org/science/wpec/sg37/Meetings/2015_May/SG37_8_LF.pdf

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$$\sum_i Y_i(Z_1, A_i) = \sum_i Y_i(Z_{CN} - Z_1, A_i)$$

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$$\sum_{A_i > \frac{A_{CN} - \bar{\nu}}{2}} Y_i(A_i) = 1$$

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This relationship is only approximately conserved. It is debatable whether it is a valid condition. Nevertheless, it is exploited in order to help conserve the other 5 relationships.

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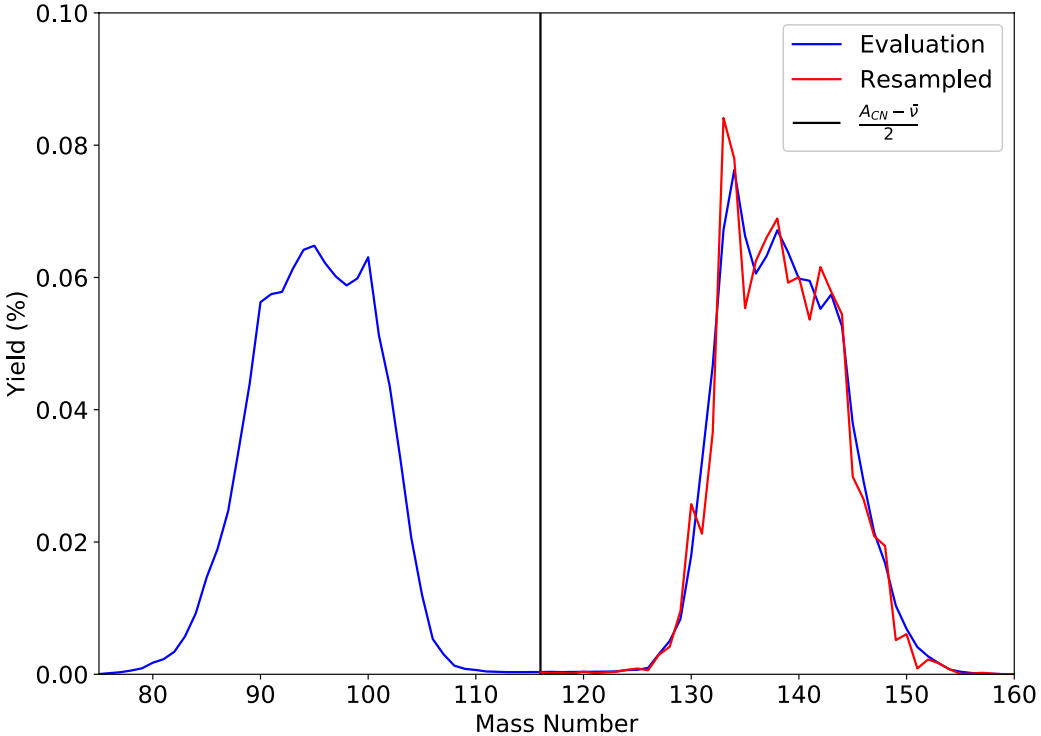
FY Covariance Matrix Generation

- The way in which a set of fission yields are resampled can be structured to conserve these relationships:
- 1) Randomly selected the "light" or "heavy" side of the fission product spectrum to resample.
- 2) Randomly select (weighted by uncertainty) a product in each A chain, resample its yield about its evaluated uncertainty.
- 3) Scale all other yields in that A chain by the same percent change.



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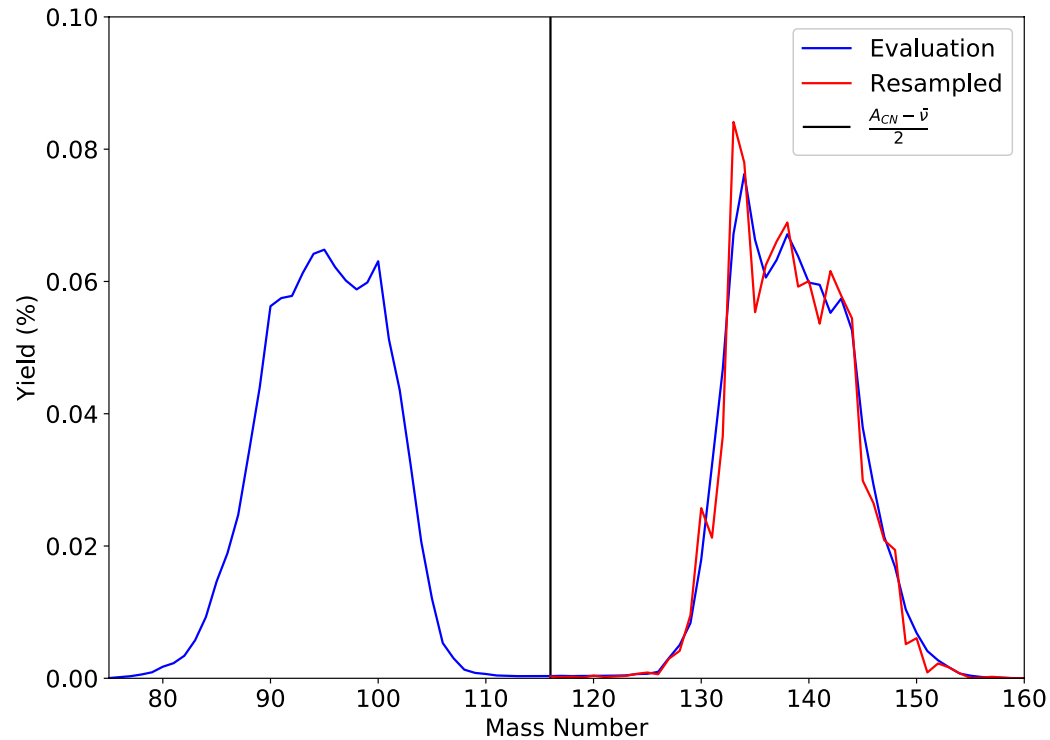


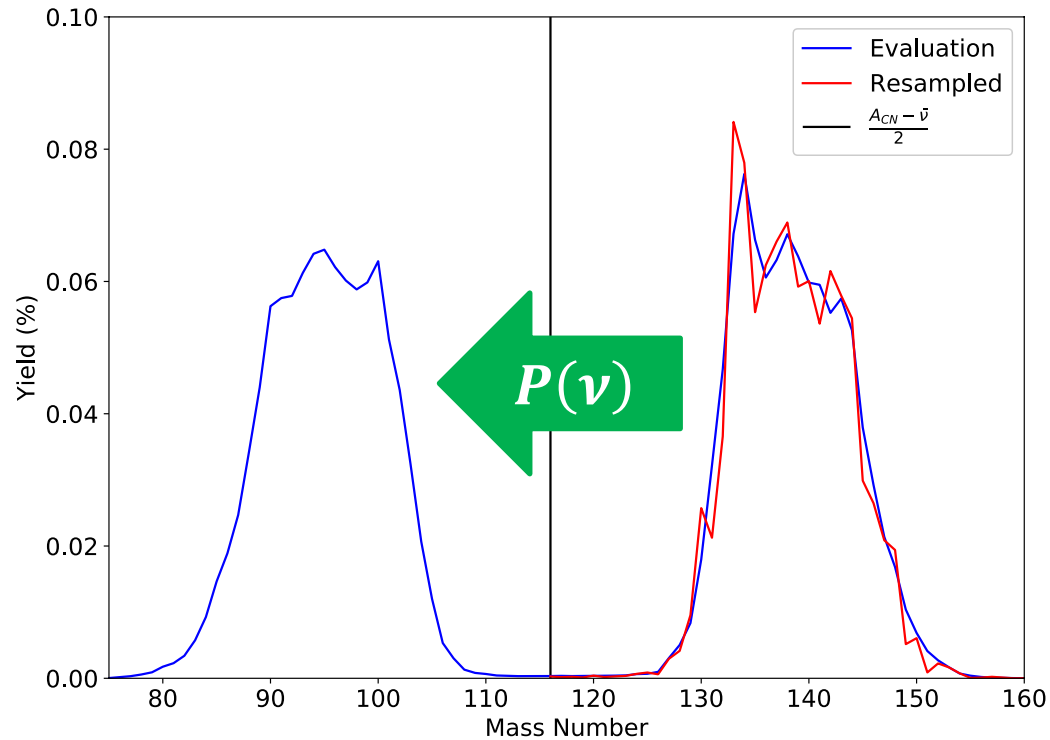
Step 3 is allowed if the Z distribution for a given A is Gaussian, which empirical data and the E&R evaluation supports [1].

FY Covariance Matrix Generation

- **4) Normalize the resampled yields such that they sum to 1.**
- **5) Generate the fission yields on the complementary side of the fission product spectrum using the neutron multiplicity of the compound system.**

$$Y_{frac}(Z_{CN} - Z, A_{CN} - A - \nu\,) = P(\nu)\, Y(Z, A)$$

$$Y(Z_{CN} - Z, A_i) = \sum_{\nu} Y_{frac}(Z_{CN} - Z, A_i)$$



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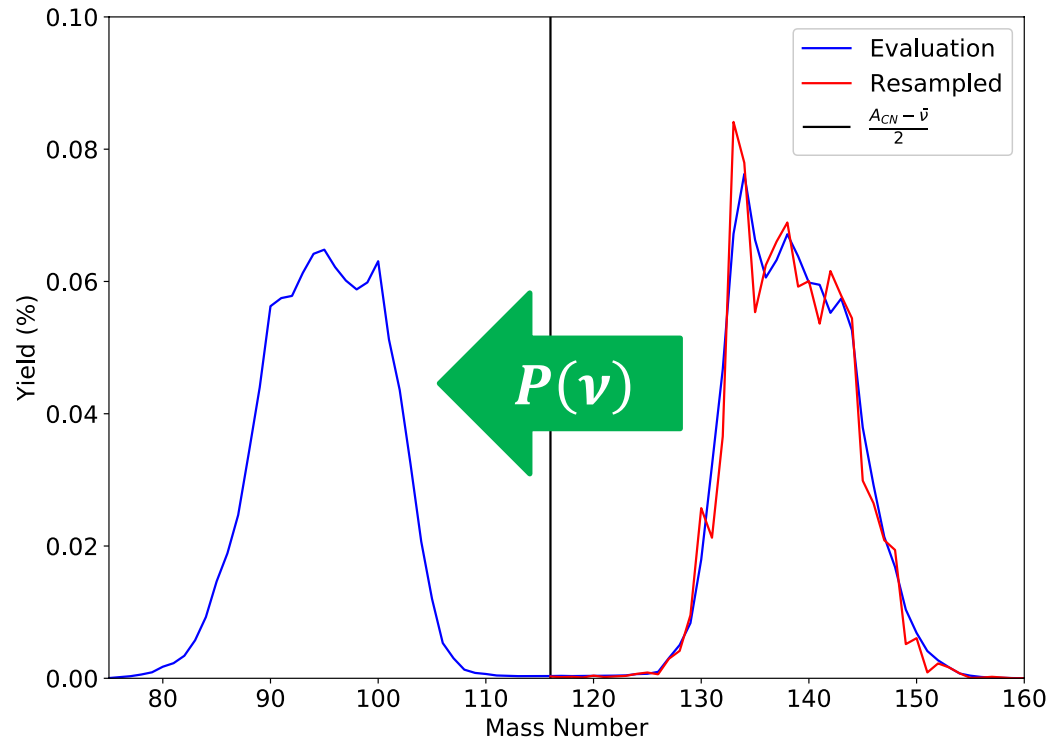
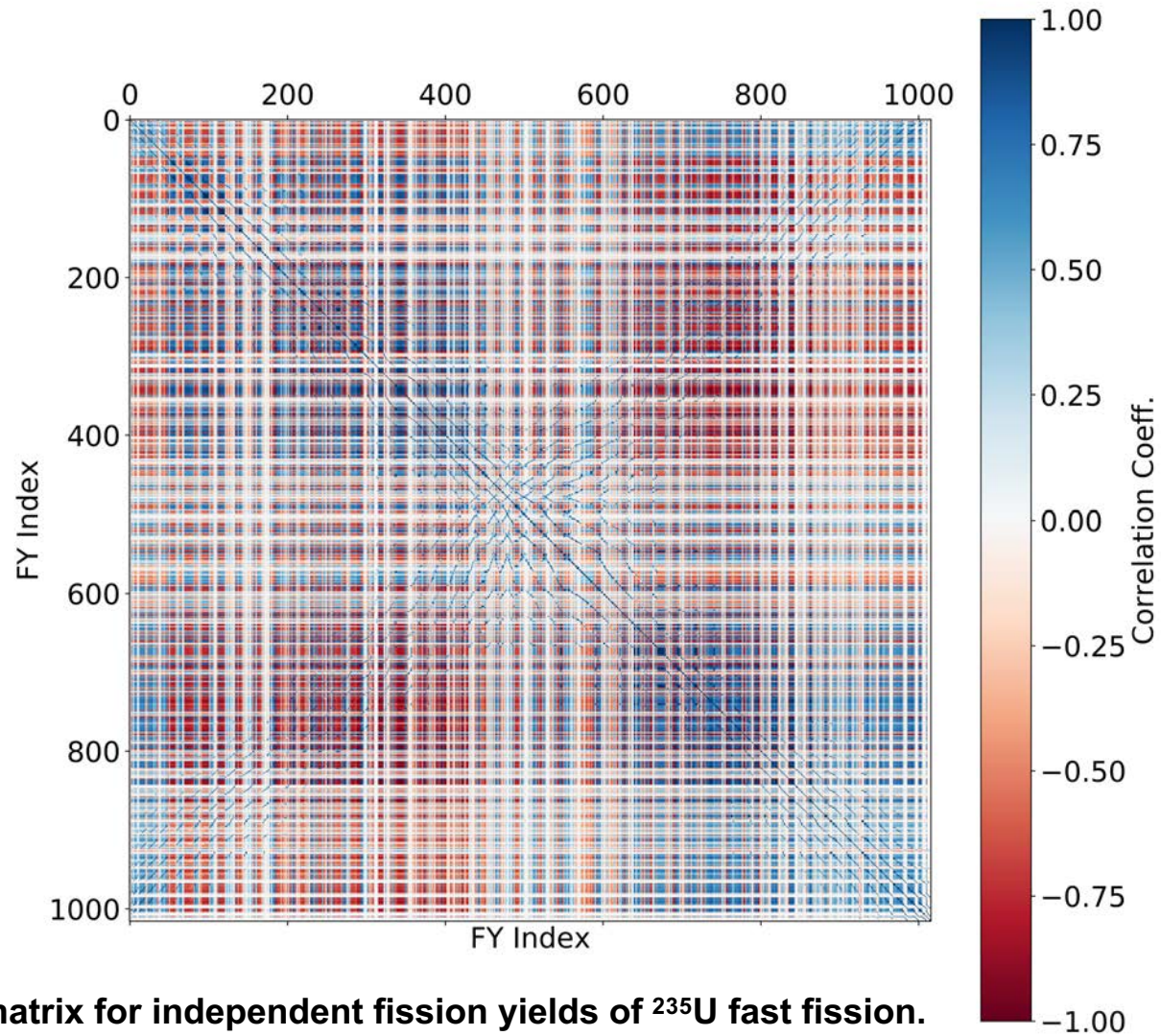


By Step 5 we've ensured all of the conservation rules are met.

FY Covariance Matrix Generation

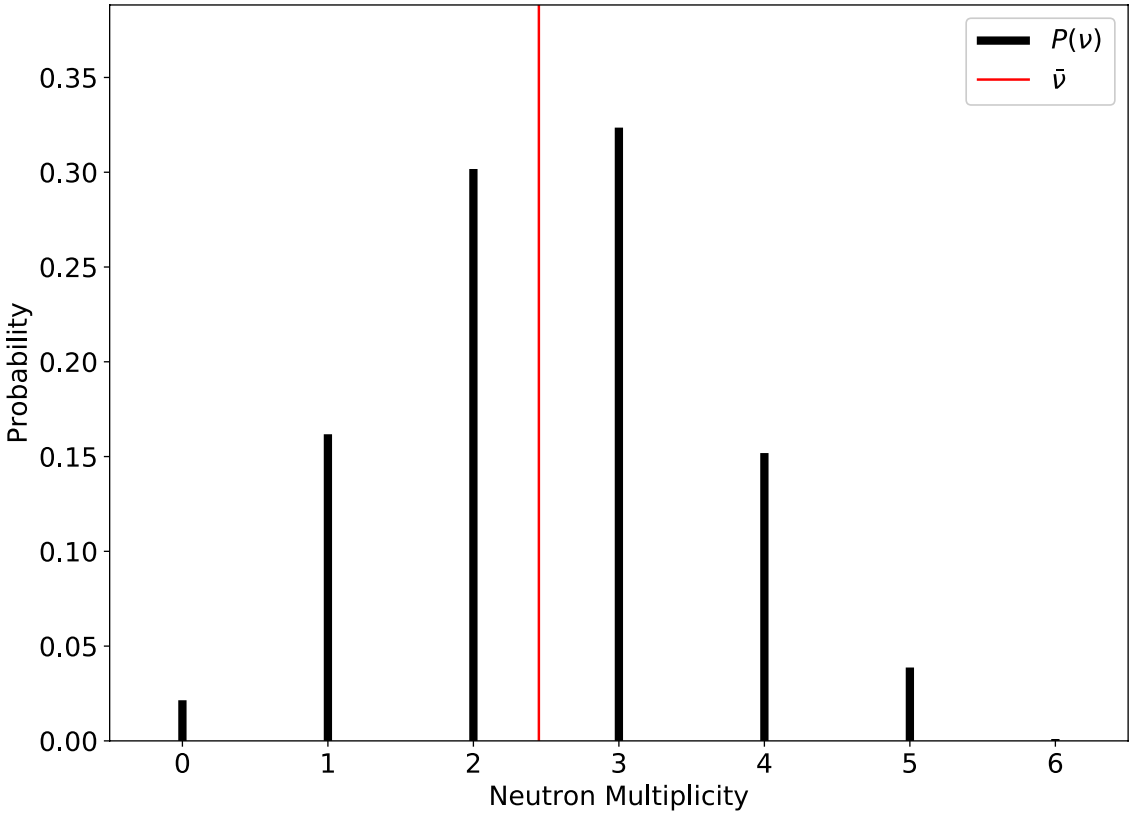
- 6) Repeat steps 1-5) N times. Select N such that statistical noise is minimized.
- 7) Calculate the resulting correlation matrix from the N trials.



Correlation matrix for independent fission yields of ^{235}U fast fission.

FY Covariance Matrix Generation

- **The England and Rider evaluation does not make any mention of the neutron multiplicity distribution used for their evaluations.**
- **Thus we are left to assume a neutron multiplicity distribution that sufficiently matches the England and Rider evaluation.**



Neutron Multiplicity for fast neutron induced fission of ^{235}U according to J.P. Lestone in LA-UR-05-0288.

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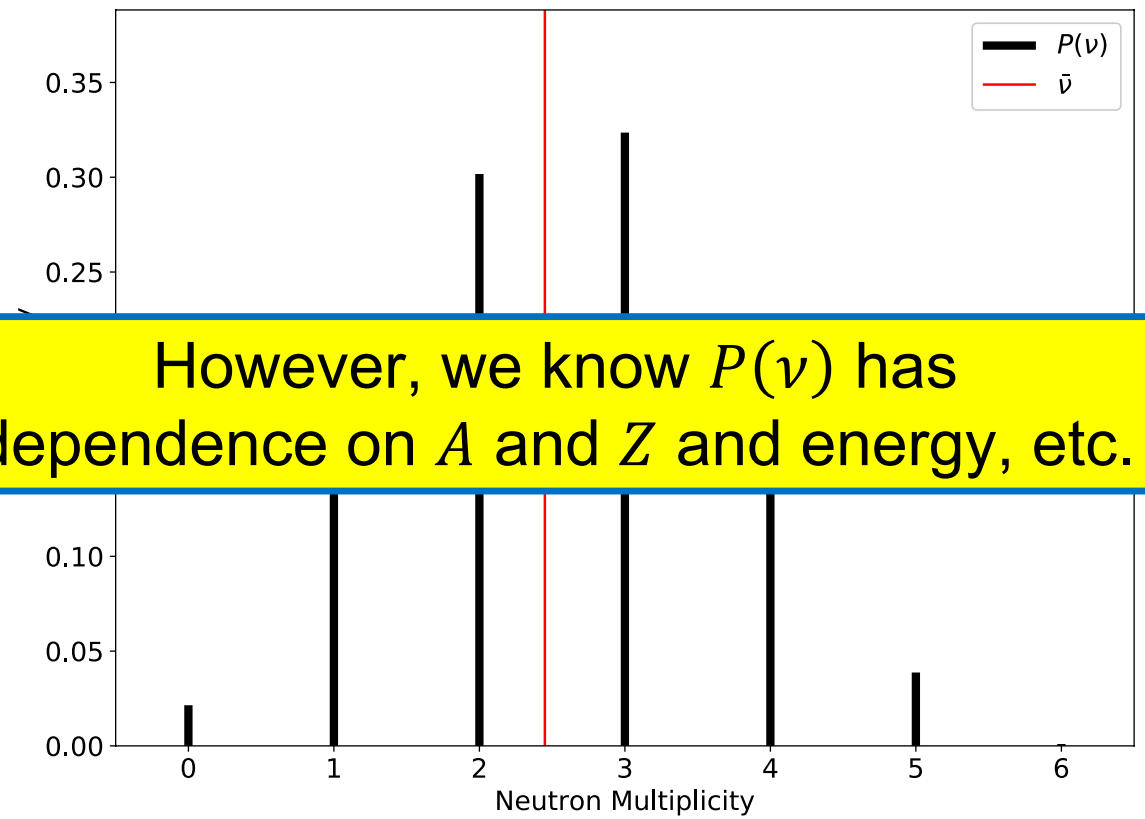


However, we know $P(\nu)$ has dependence on A and Z and energy, etc.

Neutron Multiplicity for fast neutron induced fission of ^{235}U according to J.P. Lestone in LA-UR-05-0288.

E&R Consistent $P(\nu, A)$ Data

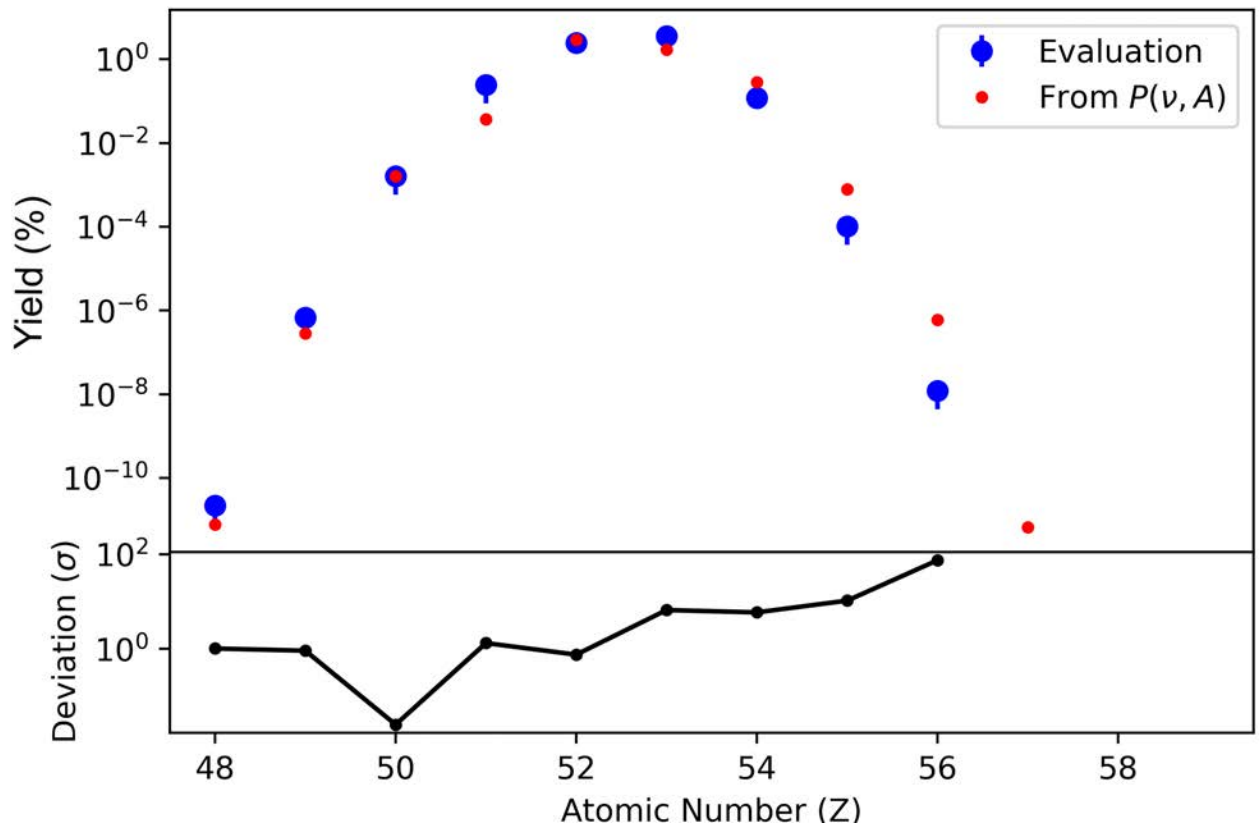
- $P(\nu, A)$ **can be fitted to the England and Rider evaluation in order to obtain the best degree of consistency.**
- **A truncated Gaussian is used to fit the shape of the** $P(\nu)$ **distribution for each** A **chain.**
- **Select** $P(\nu, A)$ **that minimizes** χ^2 **between evaluated yields and "recalculated yields",** $Y'(Z, A)$

$$Y'(Z, A) = \sum_{\nu} P(\nu, A)\, Y(Z_{CN} - Z, A_{CN} - A - \nu)$$

E&R Consistent $P(\nu, A)$ Data

<u>Example</u>:

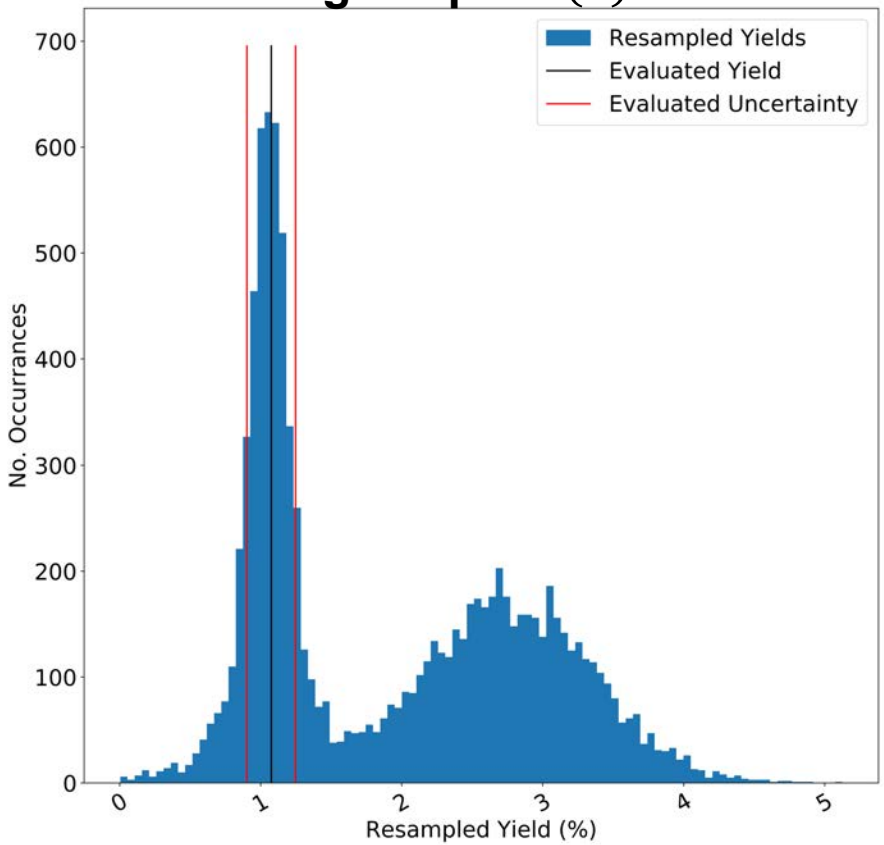
Reproduction of evaluated yields to obtain $P(\nu, 135)$ **for fast fission of ^{235}U.**



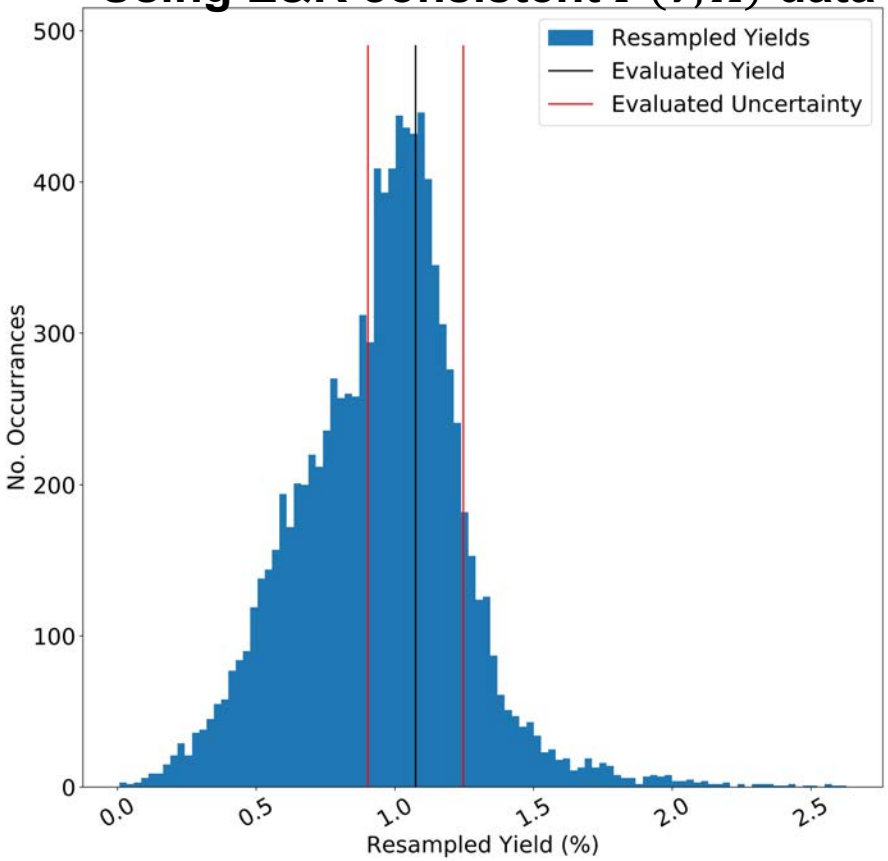
E&R Consistent $P(\nu, A)$ Data

Example: Resampled yields for ^{132}Te:

Using simple $P(\nu)$ data

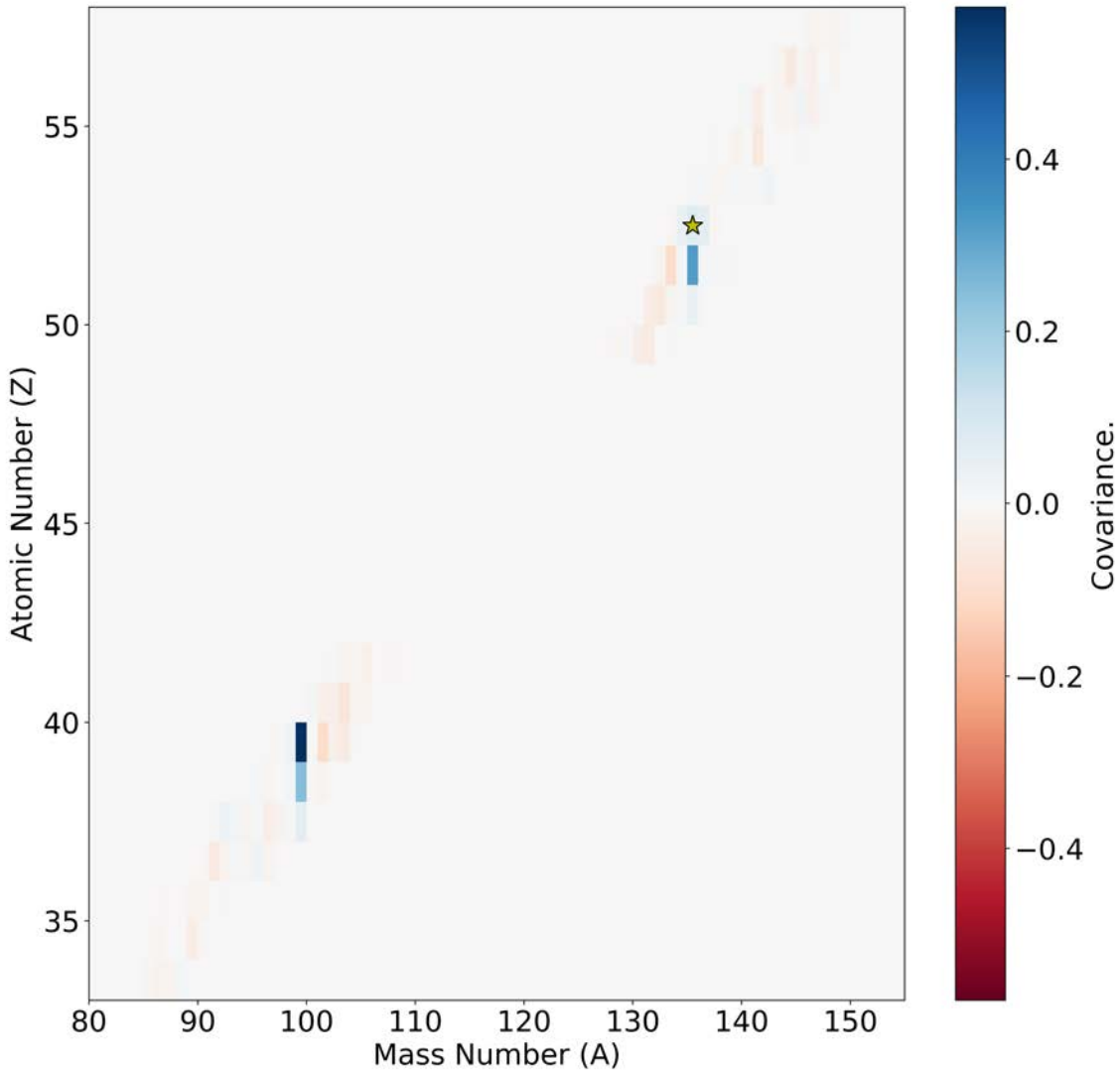


Using E&R consistent $P(\nu, A)$ data



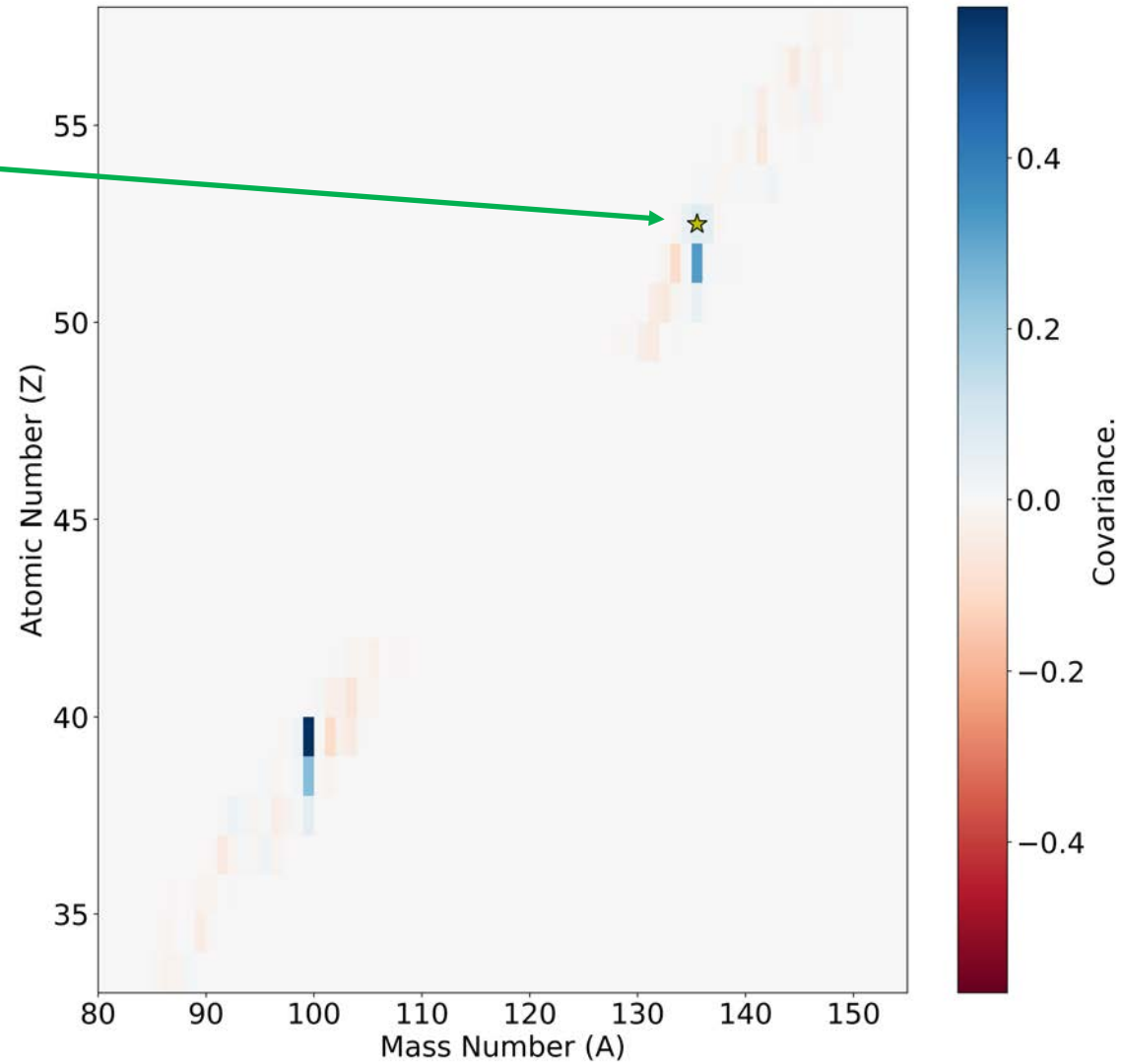
Expected Behavior

- **<u>Example:</u> ^{135}Te**
- **Presented is the covariance between independent yields as function of Z and A and that of ^{135}Te.**
- **The evaluated yield for ^{135}Te is $2.47 \pm 0.57\%$**



Expected Behavior

- **<u>Features:</u>**
- **^{135}Te is positively correlated with itself.**



Expected Behavior

- **<u>Features:</u>**
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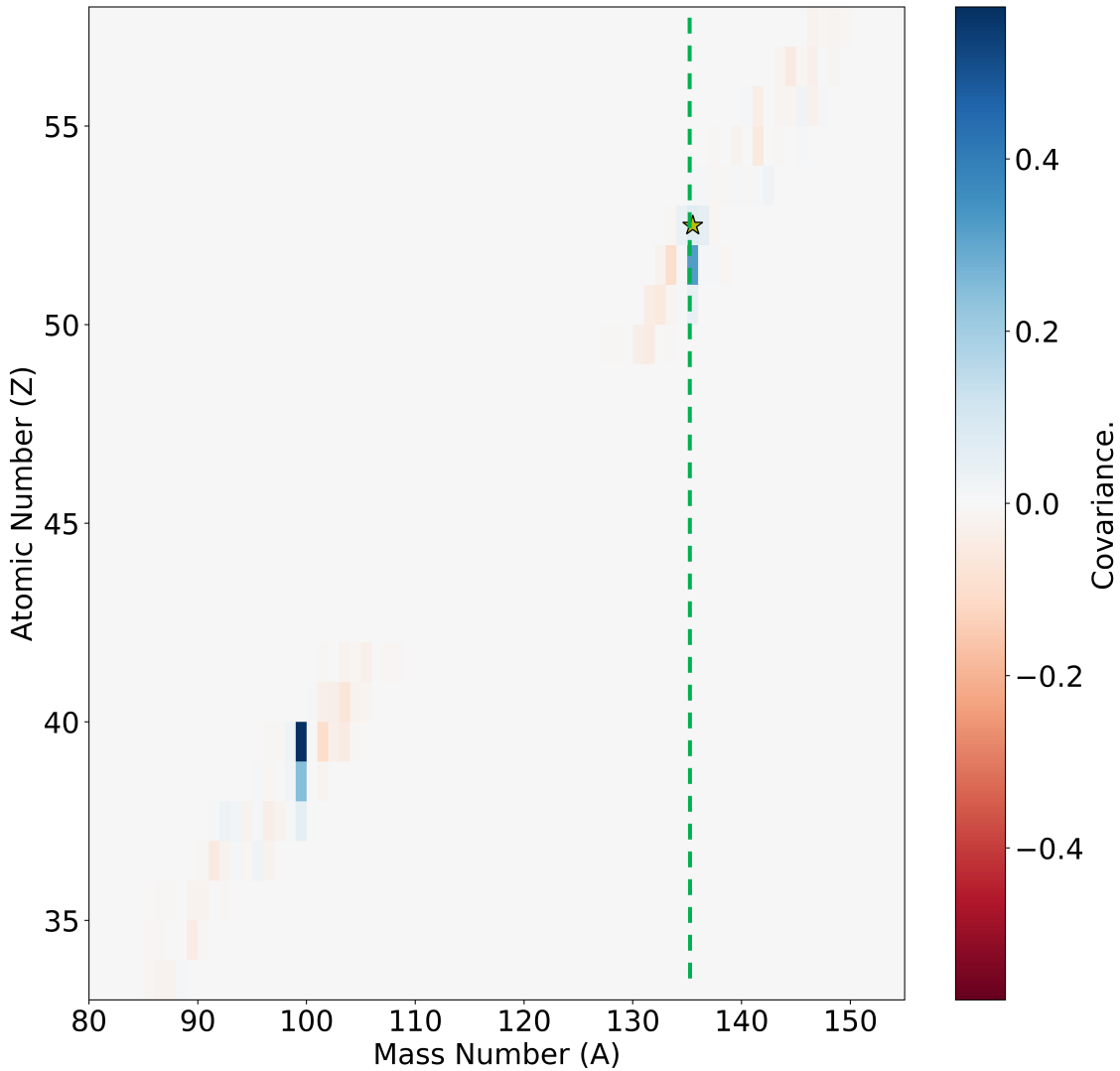
- **Products along the A chain have positive correlation.**



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- **<u>Features:</u>**
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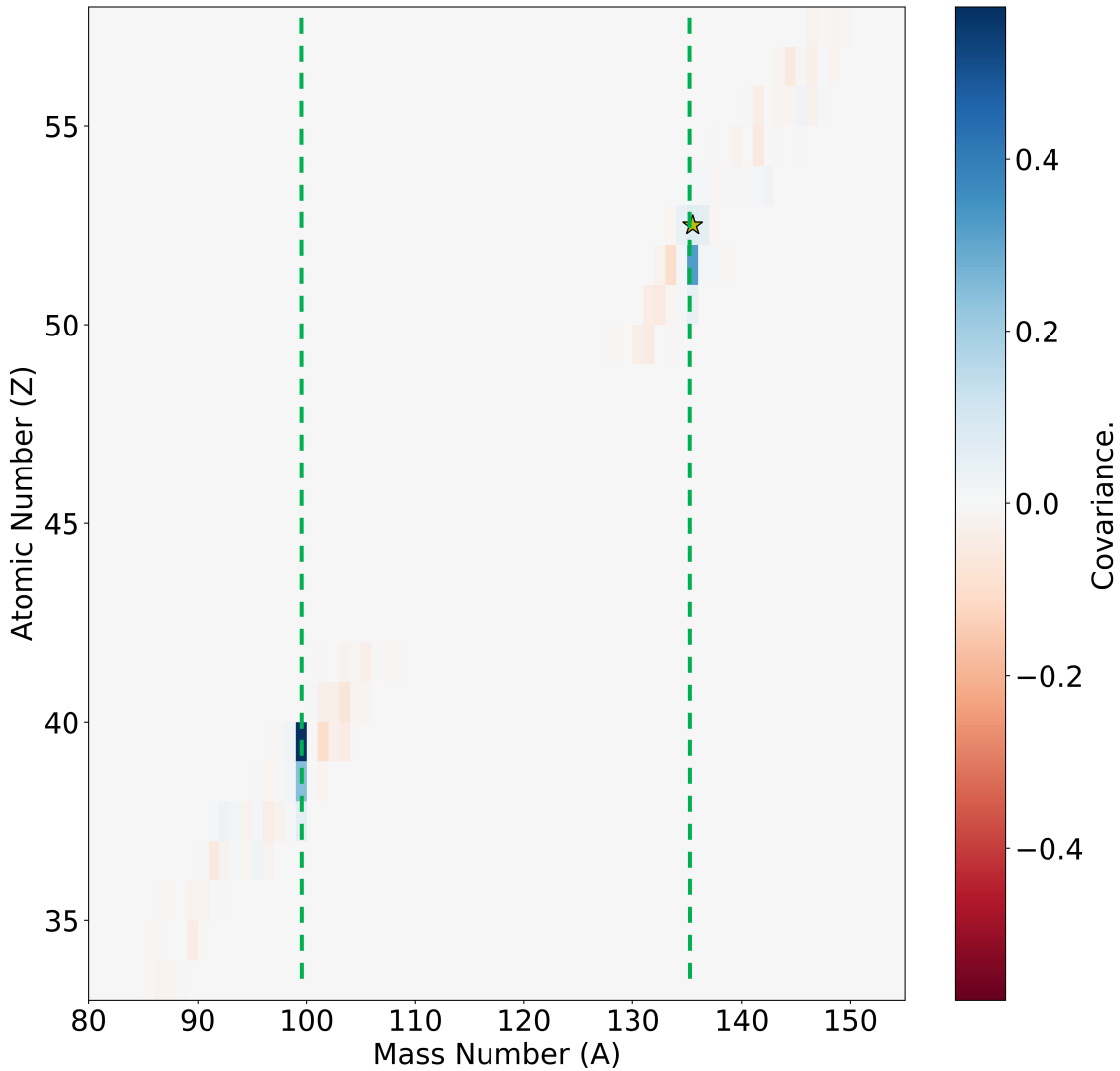
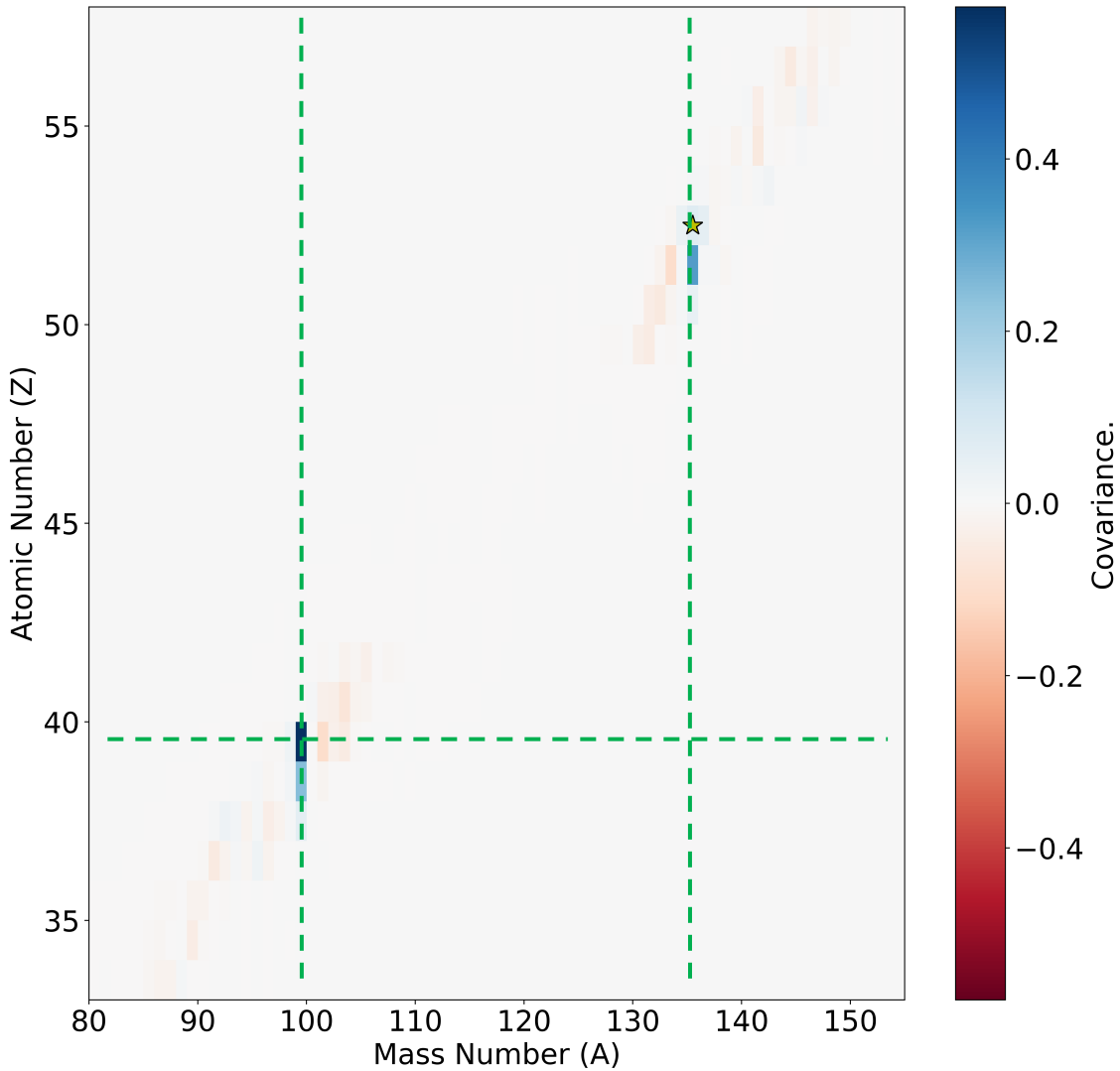
  - This positive correlation is reflected along a complementary $A = 99$ chain.



Expected Behavior

- **<u>Features:</u>**
- **^{135}Te is positively correlated with itself.**
- **Products along the A chain have positive correlation.**
  - This positive correlation is reflected along a complementary $A = 99$ chain.
- **Products along A chains that do not have complementary Z have negative correlation.**



Conclusions

- **A model-agnostic method for independent fission yield covariance matrix generation is being developed.**
- **<u>This method has been successfully applied to all 61 compound systems in the England and Rider evaluation.</u>**
- **The results demonstrate expected behavior and trends.**
- **Final results serve as an interim solution for independent fission yield covariance matrices until a new evaluation is completed.**
  - The results are publicly available at nucleardata.berkeley.edu/FYCOM
  - A peer-reviewed publication on this method has been submitted to journal.

NSSC Experience

Oral presentations at UPR 2019, 2018, Posters at UPR 2017 and UITI 2016. Won "Best National Laboratory Collaboration" at UPR 2019.

Student in 2017 and GSI in 2019 for "Nuclear Security: The Nexus Between Policy and Technology" an NSSC sponsored course at UC Berkeley.

Participant in LANL/SNL Nuclear Security and Science Summer School in 2015.

Acknowledgements

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